HYDRODYNAMIC PERFORMANCE OF WAVE-DRIVEN ARTIFICIAL UPWELLING DEVICE

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ABSTRACT: Artificial upwelling and mixing is a new technology for enhancing open ocean mariculture using nutrient-rich deep ocean water. A wave-driven artificial upwelling device was developed based on mathematical modeling and hydraulic experiments. Results of the numerical modeling are in good agreement with hydraulic experiments. The mathematical model was applied to simulate the operation of a prototype device in typical Hawaiian waves. The prototype device consists of a buoy with a water chamber 4.0 m in diameter, a flow-controlling valve, and a long tail pipe 300 m in length and 1.2 m in diameter. This device can produce an upwelling flow of 0.62 m^3/s in regular Hawaiian waves with a period of 12 s and a wave height of 1.9 m. Two analytical solutions to the simplified governing equations were also derived that provide a basis for the derivation of predictive equations, useful for preliminary engineering design and analysis.

INTRODUCTION

Natural upwelling of deep ocean water (DOW) occurs off the west coast of North America, South America, West Africa, and other coastal regions. It occurs where along-shore wind stresses push the surface seawater away from the coast, allowing the cold water from the ocean depth to upwell. This water, from depths of 300 meters and below, is rich in nutrients such as nitrate, phosphate, and silicate; is clean, being low in dissolved organic substances, suspended solids, and man-made pollutants; and is relatively free of pathological organisms.

This coastal upwelling provides a steady supply of nutrients; consequently, high fish production has been observed in these areas. In fact, while upwelling regions account for only 0.1 percent of the world's ocean, they yield roughly 44% of the fish catch (Roels et al. 1978). If nutrient-rich DOW could be distributed in the surface waters, it would open a potentially vast new source of food by dramatically increasing fish and other marine organism populations.

Land-based mariculture using DOW pumped from the ocean depths into man-made ponds and enclosures has been in existence since the 1970s on an experimental basis (Roels et al. 1978) and on a commercial basis in the United States and in Japan (Daniel 1984; Nakashima 1995). The ability of the nutrient-rich DOW to enhance the growth of fish and other marine organisms has been proven by these ventures. However, the feasibility of using DOW in the open ocean has not been proven. The major obstacles to commercial open ocean mariculture are the difficulty of bringing up the DOW to the surface and of containing it within an area of the open ocean without significant dilution.

In this research, wave-driven artificial upwelling is investigated to develop a device that can cost effectively bring DOW to the surface.

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MATHEMATICAL MODELING AND HYDRAULIC EXPERIMENTS

A device such as that shown in Fig. 1 can bring DOW to the surface by converting wave power to the kinetic energy in upwelling water. As the wave crest approaches, the flow-controlling valve of the device closes and the water column inside the device rises together with the device. As the wave descends, however, the valve opens and the water column inside the device continues its upward movement due to inertia. Therefore, when the device moves up and down in the ambient waves, the water column inside the device keeps moving upward and brings the deep ocean water to the surface.

A mathematical model has been developed to evaluate the performance of the device, which depends on how efficiently the wave power is converted into the kinetic energy of the water column inside the device. This mathematical model consists of a set of mathematical equations that describe the simultaneous movement of the water column inside the device and the device itself.

When the valve is closed, the velocity of the water column relative to the device U is zero, or

$$U = 0 \quad (1)$$

Under this condition, the equation of motion of the device takes the following form:

$$(m + m_w)\ddot{z} = -m_a\ddot{z} - b\dot{z} - \beta|\dot{z}|\dot{z} - \rho g S_w z + F_e \quad (2)$$

where m_w = mass of the water in the pipe; z = displacement of the heave of the buoy above the still water line; m = mass

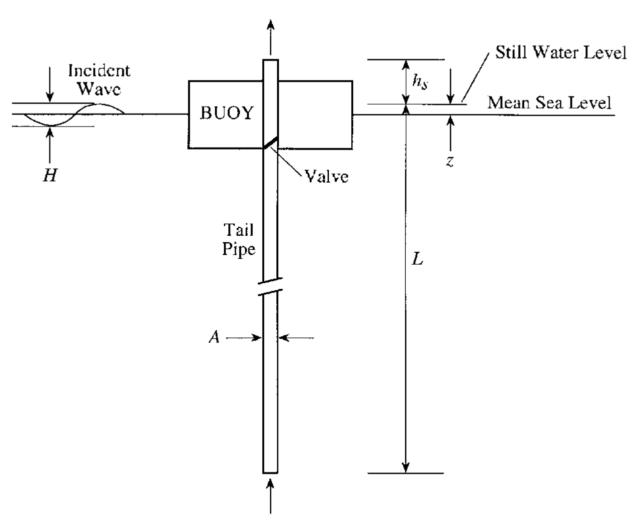

FIG. 1. Operation of Wind-Driven Artificial Upwelling Device

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of the floating system; S_w = device's cross-sectional area at the still water line; F_e = wave exciting force in the vertical direction; m_a = added mass; b = damping coefficient; and β = viscous coefficient due to the movement of the device in ambient water.

When the valve is open, the relative acceleration of the water column to the device can be determined by

$$\dot{U} + \ddot{z} + \frac{z + h_s}{L + h_s} g = 0 \quad (3)$$

In deriving (3), the dynamic pressure produced by the surface waves is ignored. This is justified, as the tail pipe of a prototype device is 300 m long; at that depth, the wave pressure is negligible.

The equation of motion of the device when the valve is open takes the form of

$$m\ddot{z} = -m_a\ddot{m} - b\dot{z} - \beta|\dot{z}|\dot{z} - \beta' U^2 - \rho g S_w z + F_e \quad (4)$$

where β' = viscous coefficient due to the movement of water inside the pipe.

Eqs. (2) and (4) are similar, except that in (2), only the mass of the floating system is considered, while in (4), the viscous effect due to relative movement of the water inside the pipe is included.

The added mass m_a, the damping coefficient b, and the exciting force F_e are important parameters describing the interaction between the wave and device. The exciting force indicates the magnitude of the external force acting on the device and is a function of incident waves. The added mass and damping coefficient indicate the extent of resistance and are functions of incident waves and device design, i.e., the dimensions of the device, tail pipe length, etc.

Because the horizontal dimension of the device is much smaller than the wave length, diffraction effects can be ignored, and the exciting force is caused mainly by the Froude-Krylor force, or $F_e = k(H/2)\sin(\omega t)$, where k is a restoring force coefficient.

According to the three-dimensional linear wave theory, values of added mass, damping coefficient, and exciting force can be determined by integration of velocity potentials over wetted surfaces (Faltinsen and Michelsen 1974).

The set of modeling equations, along with known added mass, damping coefficient, and exciting force, constitutes a general mathematical model for wave-driven artificial upwelling. A computer program was prepared to solve these equations numerically using the Runge-Kutta method (Liu and Jin 1995).

Various device designs were considered and evaluated by numerical modeling and by a series of hydraulic experiments. Hydraulic experiments were conducted in a wave basin at the Oceanographic Engineering Laboratory of the University of Hawaii (Chen et al. 1995).

First, radiation, diffraction, and free oscillation experiments were conducted to evaluate hydrodynamic coefficients. Values of added mass, damping coefficient, and exciting force determined by hydraulic experiments agree well with the theoretical results, especially when wave amplitudes are relatively small (Chen et al. 1995).

The performance of an upwelling device as shown in Fig. 1 was then investigated by using a 40 to 1 scale model in an 80 foot × 12 foot × 4 foot wave tank. The upwelling device was kept in an upright position by two fixed steel ropes. A wave maker was installed at one end of the tank to generate waves with different amplitudes and periods. At the opposite end of the tank, a plastic wave damper was installed to control wave reflection. Several wave gauges, which were connected to a data acquisition system, measured wave amplitude and period. A WVM liquid-velocity meter manufactured by Delft

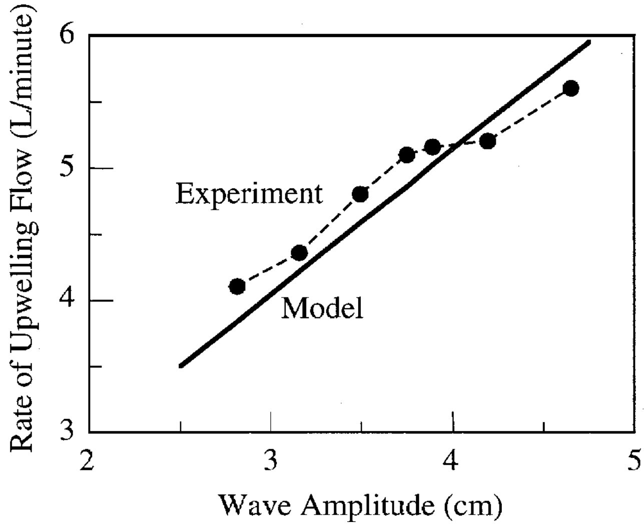

FIG. 2. Rates of Upwelling Flow Determined in Hydraulic Experiments and by Mathematical Modeling

Hydraulic Laboratory was used to measure the flow velocity at the outlet of the device. An electric circuit monitored opening and closure of the flow control valve during the experiments (Guo 1995). A storage container that was attached to the device collected the flow generated by the upwelling device; the water was then diverted to a larger container where it was weighed to determine the average rate of upwelling flow.

Performance of several designs of the device was evaluated by measuring the device movement and the upwelling flow rate under varying wave characteristics. Special attention was paid to the amplitude and period of ambient waves and to the location and orientation of the outlet. Generally, the rate of upwelling flow increases with amplitude for any given wave period and decreases with wave period for a given amplitude. Fig. 2 shows the relationship between the upwelling flow and wave amplitudes when the wave period is at 1.90 s. As shown in Fig. 2, experimental results verified the mathematical model.

FORMULATION OF SIMPLE PREDICTIVE FORMULAS

Prediction Formula Derived Based on Isaacs and Vershinskiy Analysis

Isaacs et al. (1976) and Vershinskiy et al. (1987) considered a simple scenario of wave-driven artificial upwelling in which wave-induced hydrodynamic effects on the movement of the upwelling device are negligible and the device follows exactly the sinusoidal movement of ambient waves. By excluding wave-induced hydrodynamic effects, a constant hydrostatic pressure head would act on the water column; thus, the velocity of the water column will change linearly when the flow controlling valve is open. Since there is no friction between the device and the water column, the valve is open when the acceleration of the water column is equal to or greater than that of the device, and closed when the velocity of the water column inside the device is equal to that of the device (Fig. 3).

The volume of upwelling discharge per wave period is the volume of upward movement of the water in the tail pipe that takes place when the valve is open, or

$$W = A\int_{t_1}^{t_2} \Delta\nu \, dt = A\int_{t_1}^{t_2} \left(\int_{t_1}^{t_2} a(t)\, dt - \frac{H}{2}\omega \cos(\omega t) \right) dt \quad (5)$$

where $a(t)$ = acceleration of the water column inside the tail pipe, which is caused by the water pressure acting on the bottom of the tail pipe; H = wave height; and ω = wave frequency.

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Generally, the pressure consists of static pressure and dynamic pressure (Fig. 1). By ignoring the dynamic pressure, the acceleration of the water column inside the tail pipe can be determined by the following equation:

$$a(t) = \frac{h_s}{L + h_s} g \tag{6}$$

By definition, t_1 and t_2, which denote the time when the valve is open or closed, can be determined by satisfying the conditions

$$a(t) = -\frac{H}{2} \omega^2 \sin \omega t \tag{7}$$

$$\int_{t_1}^{t_2} a(t)\, dt = \int_{t_1}^{t_2} -\frac{H}{2} \omega^2 \sin (\omega t)\, dt \tag{8}$$

This volume of upwelled flow can be determined graphically as the shaded area between velocity profiles of the water column and the device (Fig. 3). This area can be further approximated by taking the velocity profile of the device as two straight lines; thus, the shaded area becomes a triangle (Fig. 4).

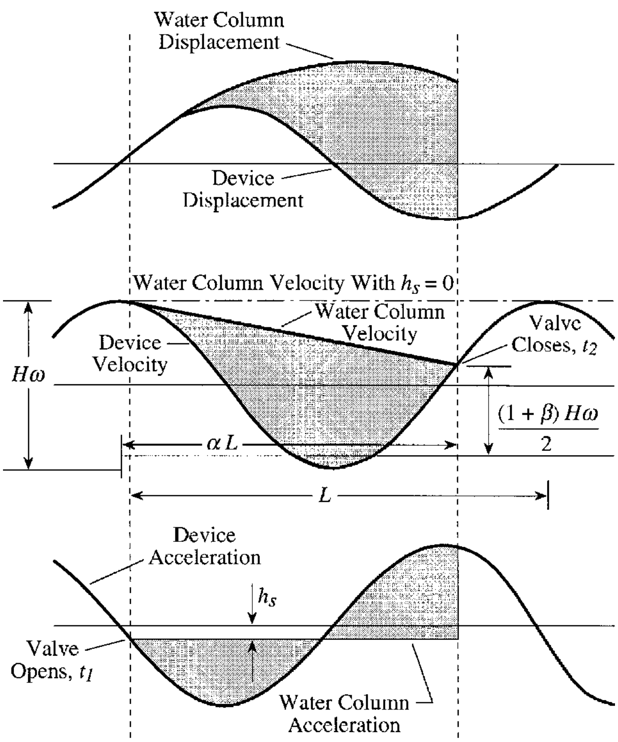

FIG. 3. Hydrodynamic Response of Device and Water Column inside Tail Pipe to Ambient Waves [after Isaacs et al. (1976)]

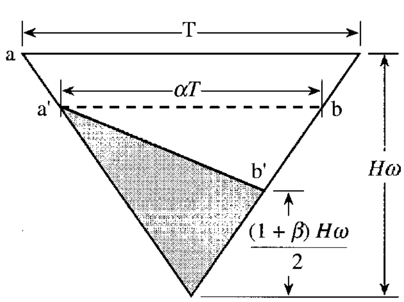

FIG. 4. Approximation of Volume of Upwelling Flow

$$W \approx A \left[\frac{1}{2} T \cdot H\omega - \frac{1}{2} T \cdot H\omega \cdot (1 - \alpha^2) - \frac{1}{2} T \cdot H\omega \left(\alpha^2 - \alpha \frac{1 + \beta}{2} \right) \right]$$
$$= A \left[T \frac{H}{2} \omega \cdot (1 + \beta)\alpha \right] = \pi\alpha(1 + \beta) A \frac{H}{2} \tag{9}$$

So

$$\bar{Q} = \frac{W}{T} = \pi \frac{AH}{2T} \alpha(1 + \beta) \tag{10}$$

Both α and β denote the hydrodynamic performance of the upwelling system. The coefficient of α represents the ratio of the oscillation displacement of the device to the wave amplitude. When the wave period is large and the device nearly follows the wave motion, point a′ in Fig. 4 would approach point a and, thus, α would approach a value of 1. The coefficient β depends on the hydrostatic pressure acting on the bottom of the water column. If the hydrostatic pressure is zero, point b′ in Fig. 4 would approach point b and β would also approach 1.

The Isaacs and Vershinskiy analysis is conducted with the assumption that the device follows exactly the sinusoidal movement of ambient waves. Therefore, it requires that the value of α remains unity. The value of β depends on the height of the outlet above the still water level, or h_s (Fig. 1). The maximum upwelling flow rate occurs when $h_s = 0$ and there is no hydraulic pressure acting on the bottom of the water column. Under these condition, (10) becomes

$$\bar{Q} = \frac{W}{T} = \pi \frac{AH}{T} \tag{11}$$

For ocean wave with periods of 8–12 s, the movement of the device deviates significantly from that of the ambient wave. Under these conditions, the Isaacs and Vershinskiy formula, (11), is not valid. A linear analysis as presented below was conducted to evaluate α when the device does not follow the movement of the ambient waves.

Linear Prediction Formula

A somewhat more detailed analysis can be conducted based on a linear approximation of (2) and (4). In this case, the displacement of the upwelling device will no longer be the same as that of the ambient wave.

Eqs. (2) and (4), which describe the motion of the device when the flow controlling valve is open or closed, are different due to the terms of the inertia force $m_w \ddot{z}(t)$ and the damping force $\beta' U^2(t)$. The inertia force exists only when the valve is closed, and the damping force exists only when the valve is open.

Eqs. (2) and (4) can be replaced by a single linear equation with the following assumptions:

1. The mass of the upwelling system remains constant during heaving movement, or

$$m = m_d + \delta m_w + m_a = m_d + \delta\rho\pi\, d^2 L/4 + a m_d \tag{12}$$

where δ = coefficient between 0 ~ 1, since the water column would move together with the device when the flow controlling valve is closed, occurring only for part of a wave period. The coefficient a = added mass coefficient; and the coefficient d = diameter of the tail pipe.

2. The damping coefficient is a constant during heaving movement, or

$$c = b + \beta |\dot{z}(t)| + \beta' \frac{U(t)^2}{\dot{z}(t)} \approx \text{const} \tag{13}$$

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With these assumptions, the governing equations becomes

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = |F_e|\sin \omega t \quad (14)$$

where k = restoring force coefficient.

The displacement of the device heave movement $z(t)$ can be readily determined by solving (14), or

$$z(t) = \left[\frac{|F_e|\sin[2\pi ft - \phi(f)]}{k\sqrt{[1 - (2\pi f)^2 m/k]^2 + [fc/k]^2}}\right] \quad (15)$$

Eq. (15) can be expressed in terms of two dimensionless terms of damping ratio ζ and frequency ratio $\xi = f/f_n$ as

$$z(t) = \left[\frac{|F_e|\sin[2\pi ft - \phi(f)]}{k\sqrt{(1 - \xi^2)^2 + (2\zeta\xi)2}}\right]^2 \quad (16)$$

The damping ratio ζ is defined as

$$\zeta = \frac{c}{2\sqrt{km}} \quad (17)$$

and the frequency ratio between wave frequency f and undamped natural frequency f_n is defined as

$$\xi = \frac{2\pi f}{\sqrt{k/m}} \quad (18)$$

Note that the frequency of an incident monochromatic wave $f = \omega/2\pi = 1/T$ where T is the wave period.

With these definitions, the phase angle between heave of the device and wave is

$$\phi(f) = \tan^{-1}\left[\frac{2\zeta\xi}{1 - \xi^2}\right] \quad (19)$$

By assuming the sine function in (16) to be one, the amplitude of the device heave movement of the device is shown as

$$z_0 = \left[\frac{|F_e|/k}{\sqrt{(1 - \xi^2)^2 + (2\zeta\xi)^2}}\right] \quad (20)$$

From (16), a magnifying factor μ can be expressed as follows:

$$\mu = \left[\frac{1}{k\sqrt{(1 - \xi^2)^2 + (2\zeta\xi)^2}}\right] \quad (21)$$

The velocity of the device heave movement can be derived by differentiating (16)

$$\dot{z}(t) = \left[\frac{2\pi f|F_e|\cos[2\pi ft - \phi(f)]}{k\sqrt{(1 - \xi^2)^2 + (2\zeta\xi)^2}}\right] \quad (22)$$

When the outlet of the device is located at or below the still water level of the ocean, h_s equals zero (Fig. 1). Under this condition, the hydrostatic pressure is zero, and there is only heave acceleration acting on the water column inside the pipe. Therefore, the pump discharge rate in one heave period of the device is

$$Q(t) = [\dot{z}_0 - \dot{z}(t)]A$$

$$= \left[\frac{2\pi f|F_e|\{1 - \cos[2\pi ft - \phi(f)]\}}{k\sqrt{(1 - \xi^2)^2 + (2\zeta\xi)^2}}\right]\frac{\pi}{4}d^2 \quad (23)$$

where $\dot{z}_0$ = maximum heave velocity, which is the relative velocity of the water column when the valve is open (Fig. 2).

The average discharge rate can be determined by this simple linear analysis as

$$\bar{Q} = \frac{1}{T}\int_{t_1}^{t_1+T} dt \cdot Q(t) = \frac{1}{T}\int_{t_1}^{t_1+T} dt \cdot [\dot{z}_0 - \dot{z}(t)]A = \frac{2\pi z_0 A}{T} \quad (24)$$

Therefore, the average discharge rate is

$$\bar{Q} = \frac{2\pi z_0 A}{T} = \left[\frac{2\pi|F_e|/k}{\sqrt{(1 - \xi^2)^2 + \xi^2}}\right]\frac{A}{T} \quad (25)$$

Note that by considering only the Froude-Krynov wave force, $|F_e| = kH/2$, and by substituting (12), (17), and (18) into (25), the average flow rate of the artificial upwelling device becomes

$$\bar{Q} = \left[\frac{1}{\sqrt{[1 - 2\pi f/\sqrt{k/(m_d + \delta\rho\pi\, d^2L/4 + am_d)^2}]^2 + [fc/k]^2}}\right] \cdot (\pi HA/T \quad (26)$$

Note that in (26), the length of the pipeline is contributing to the mass of the system, which, in turn, reduces the natural frequency of the device and makes a greater magnifying factor in lower frequency waves.

When the wave frequency f is very small, (26) become the Isaacs and Vershinskiy equation, (11). When the wave frequency f gets larger, however, the bracket in (26) would become greater than 1 first and then approach 0.

HYDRODYNAMIC PERFORMANCE OF PROTOTYPE DEVICE

Various prototype device designs were considered and evaluated. The one selected has the following characteristics:

- Diameter of the buoy: D = 4.0 m
- Draft of the buoy: d_r = 2.0 m
- Length of the tail pipe: L = 300 m
- Diameter of the tail pipe: d = 1.2 m
- Cross-sectional area of the tail pipe: A = 1.16 m^2
- Outlet height above the water level: h_s = 0.5 m
- Still water area: s_w = 12.5 m^2

This prototype design was evaluated using regular Hawaiian waves with the following characteristics:

- Wave period: $T = 8 \sim 12$ s
- Wave height: H = 1.9 m

Values of hydrodynamic coefficients were determined as follows:

- Damping coefficient: c = 650 kg/s
- Added mass coefficient: a = 1.07
- Restoring force coefficient: $k = C\rho g S_w$ = 500,000 kN/m (efficient coefficient C is taken to be 0.97)
- Water column mass coefficient: δ = 0.1 (because valve is closing for 1/10 duration)

Under the above design conditions, the terms inside the bracket in (26) take a value of about 1.2, or

$$\bar{Q} = \frac{1.2\pi HA}{T} \quad (27)$$

Fig. 5 shows the performance of this prototype device in regular Hawaiian waves as calculated by the numerical model, by the Isaacs and Vershinskiy formula, and by the linear formula. All three methods indicate that the rate of upwelling flow produced by the device decreases as the wave period increases. When the wave period is 12 s, the rate of upwelling flow as calculated by the numerical model is about 6.2 m^3/s.

The Isaacs and Vershinskiy formula, (11), was formulated by assuming the device follows the sea surface exactly. It gives satisfactory results only when the wave period is large or the

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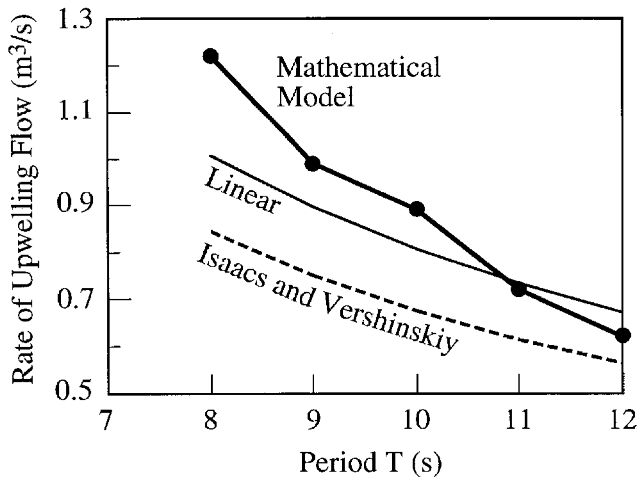

FIG. 5. Performance of Prototype Device Simulated by Mathematical Model and by Predictive Formulas

frequency of the wave is small. For regular Hawaiian waves with periods ranging between 8 and 12 s, the Isaacs and Vershinskiy formula significantly underestimates the rate of upwelling flow.

The linear formula was formulated by linearizing the governing equation. It was achieved by assuming the mass of the upwelling system and the damping coefficient to be constant regardless of whether the flow controlling valve is open or closed. With the given device and wave characteristics, the linear formula takes the form of (27). The linear formula gives a much better prediction than the Isaacs and Vershinskiy formula.

CONCLUDING REMARKS

Operating in regular Hawaiian waves, a wave-driven artificial upwelling device that consists of a buoy, a flow-controlling valve and a long tail pipe can bring up more than 0.5 m^3/s of deep ocean water to the surface.

Vershinskiy et al. (1985) reported that their calculated upwelling flow rate was about 30% less than the flow rate measured in a field experiment. This study found that the underestimation in the upwelling flow prediction by the Isaacs and Vershinskiy formula is due to its assumption that the device follows exactly the wave movement. Usually, the amplitude of the device heave movement is larger than that of the ambient wave; the difference is significant when the wave period is small. Therefore, the Isaacs and Vershinskiy formula gives satisfactory predictions only when the period of the ambient waves is large.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = cross-sectional area of tail pipe;
a = added mass coefficient;
$a(t)$ = acceleration of water column;
b = damping coefficient;
C = constant damping coefficient;
D = diameter of buoy;
d = diameter of tail pipe;
d_r = draft of buoy;
F_e = wave exciting force in vertical direction;
f = wave frequency;
f_n = undamped natural frequency of device;
g = acceleration of gravity;
H = incident wave amplitude;
h_s = length of pipe above still water level;
k = restoring force coefficient;
L = length of tail pipe under still water level;
m = mass of floating system;
M_a = added mass of upwelling system;
m_w = mass of water in pipe;
Q = flow rate;
S_w = water line area;
T = wave period;
t = time;
t_1 = time when valve is open;
t_2 = time when valve is closed;
U = relative velocity of water column to device;
W = volume of upwelling flow per wave period;
z = displacement of heave of buoy above still water line;
α = amplification coefficient;
β, β' = viscous coefficients;
$\Delta v(t)$ = difference of velocities;
ζ = damping ratio;
μ = magnifying factor;
ξ = frequency ratio f/f_n;
ρ = water density;
ϕ = phase angle between heave and wave; and
ω = wave frequency.

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