Multi-User Gain Maximum Eigenmode Beamforming, and IDMA

Peng Wang and Li Ping
City University of Hong Kong

Contents

For detail see

Peng Wang and Li Ping, “On multi-user gain in MIMO systems with rate constraints,” *IEEE GlobeCom*, Washington, DC, USA, Nov. 26-30, 2007.

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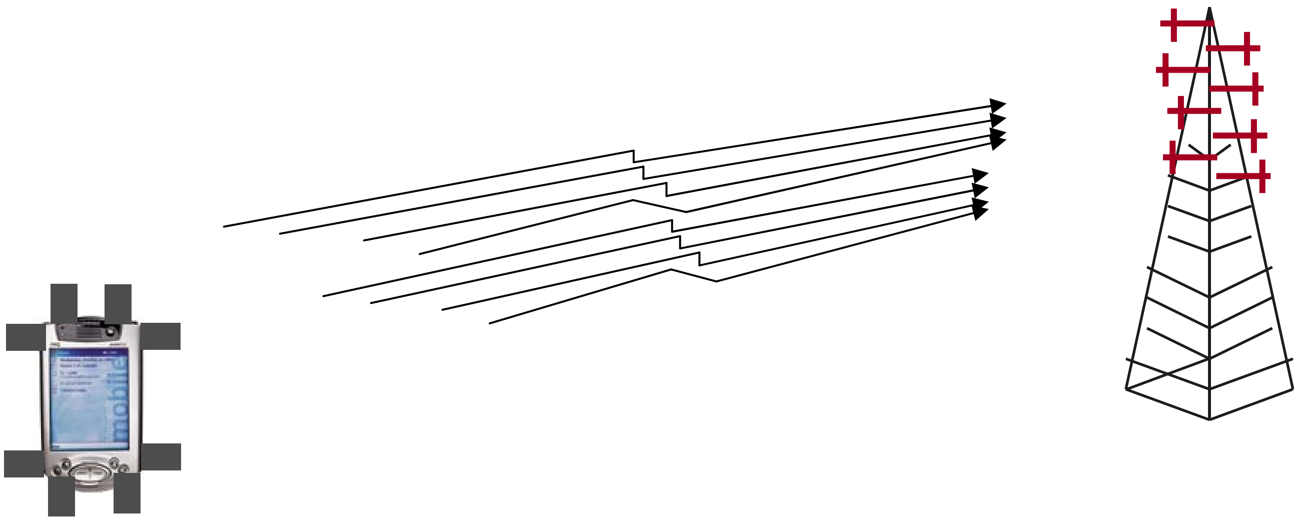
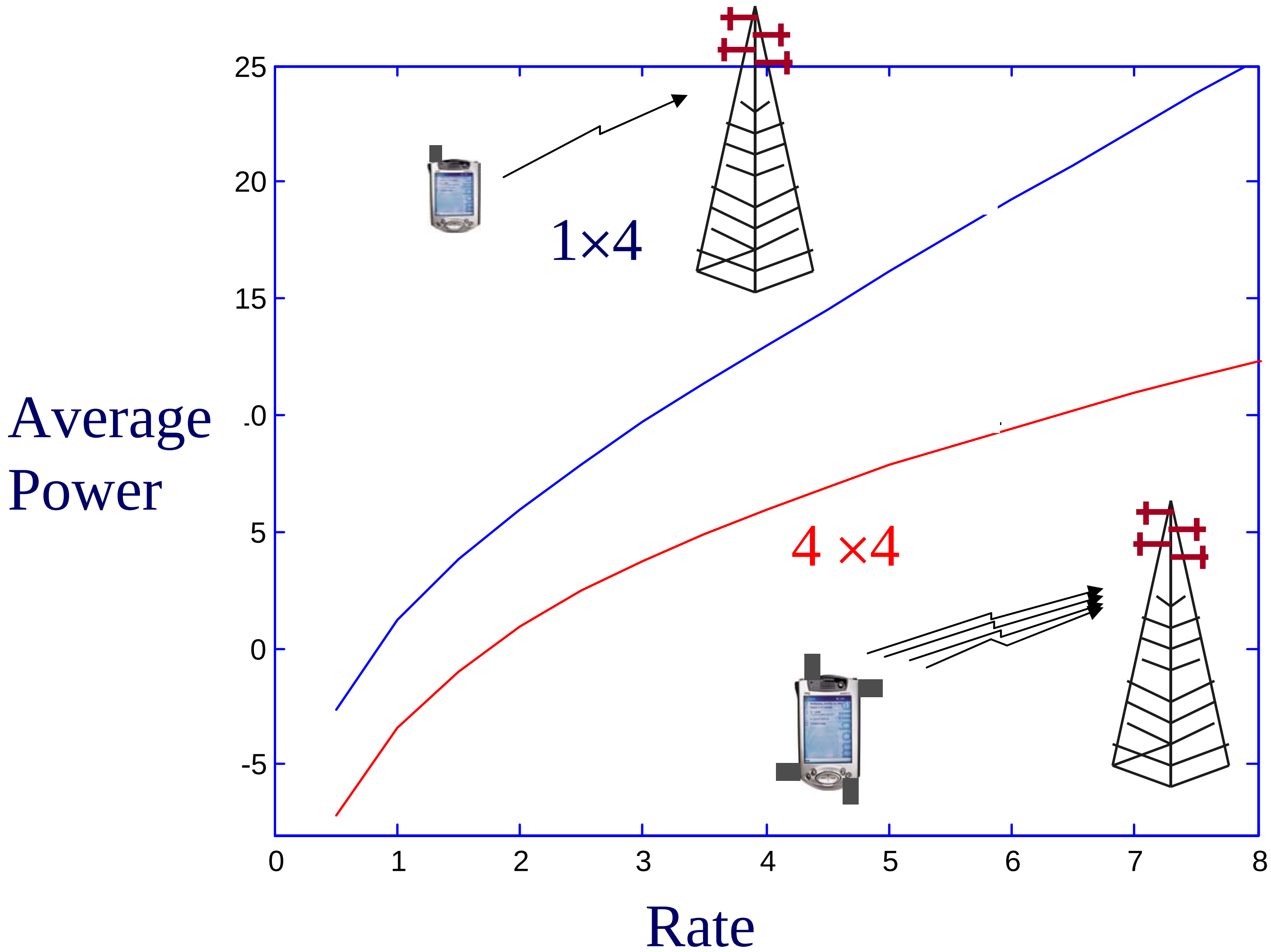
A 8×8 MIMO System



Great performance, but who wants to buy such a bulky handset?

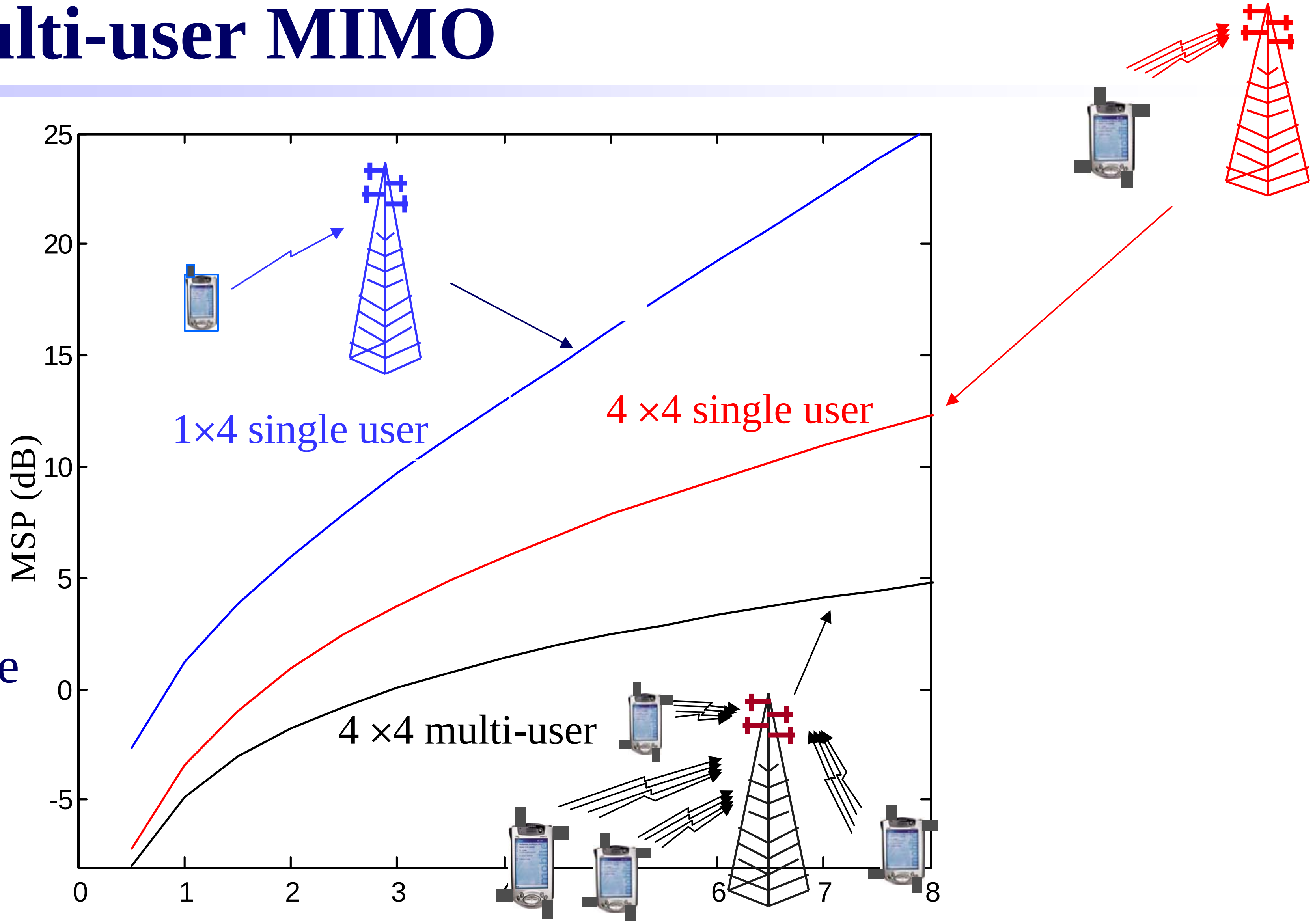
Also, performance may be seriously affected by imperfect feedback and correlation among antennas.

Advantage of MIMO



Can we do better?

Multi-user MIMO



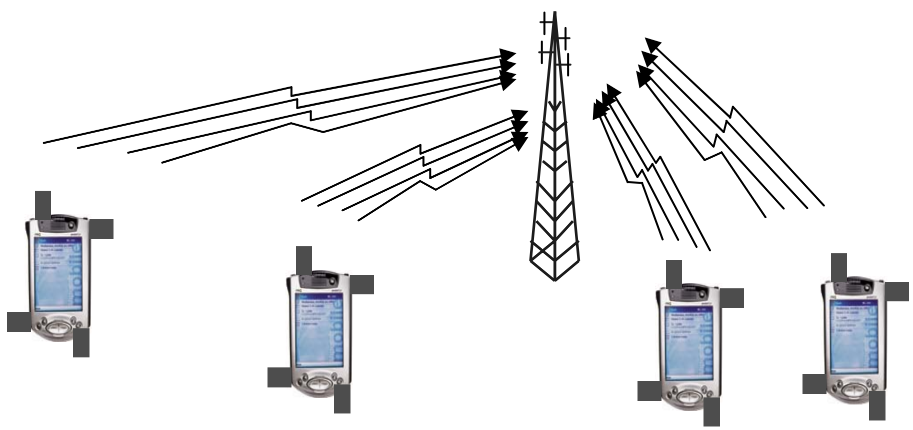
Average Power

Rate

The Problems with Multi-user MIMO

Optimized MIMO are very complicated, involving

- decoding order optimization,
- correlation matrix optimization,
- singular value decomposition (SVD),
- eigenmode water filling.



The Focus of This Talk

- What are the advantages of multi-user MIMO?
- Are there simple realization options for multi-user MIMO?

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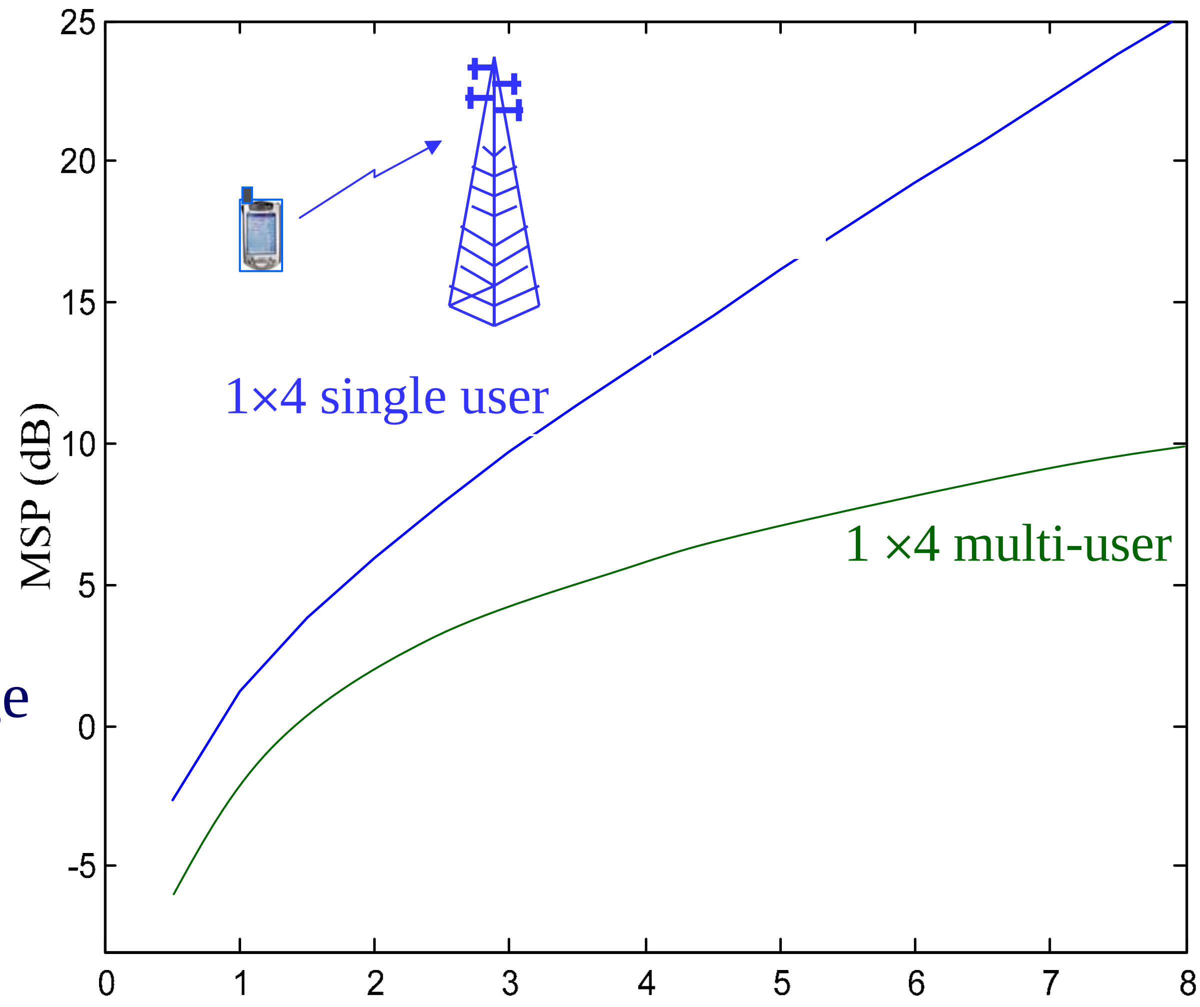
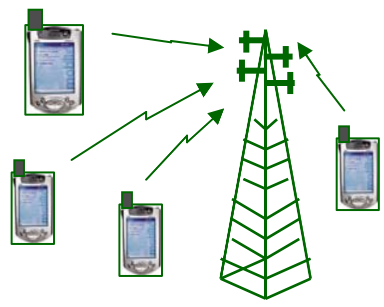
Multi User Gain

We know that power saving can be achieved by multi-user concurrent transmission. But how much is the multi-user gain (without scheduling)?

Multi-user Gain for SIMO

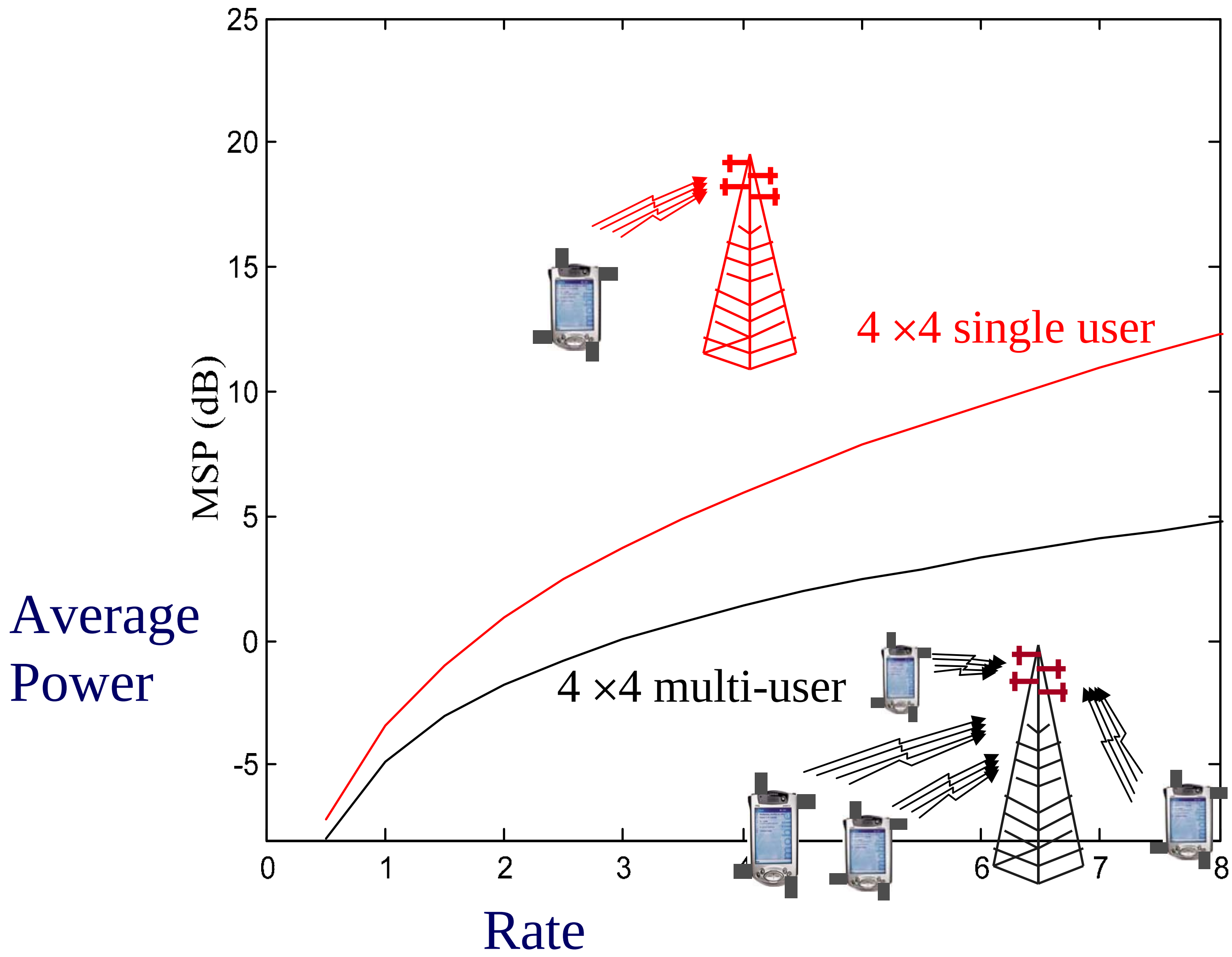




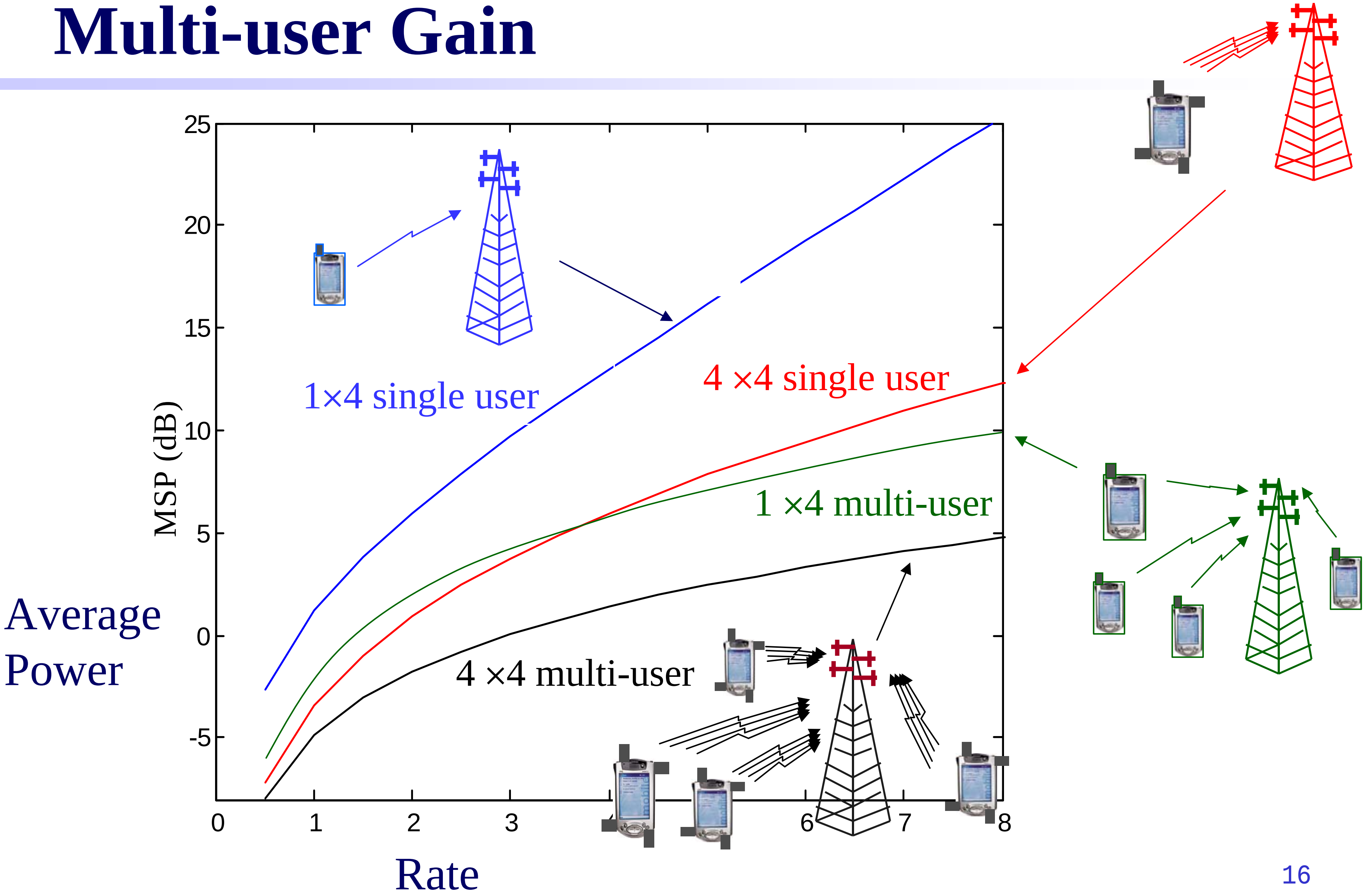
Average Power

Rate

Multi-user Gain for MIMO



Multi-user Gain



What is the reason of multi-user gain?

For detail, see

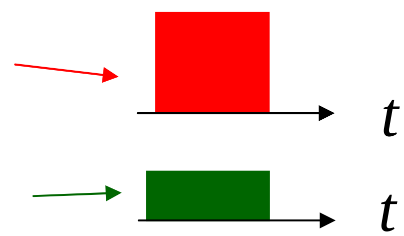
Peng Wang, Jun Xiao, and Li Ping, "Comparison of orthogonal and non-orthogonal approaches to future wireless cellular systems," *IEEE Vehicular Technology Magazine*, vol. 1, no. 3, pp. 4-11, Sept. 2006.

Stripping Decoding

- Decode ***x***'' by treating ***x***' as noise $$R/2 = \log_2(1 + \frac{q''}{N+q'})$$
- Decoding ***x***' $$R/2 = \log_2(1 + \frac{q'}{N})$$

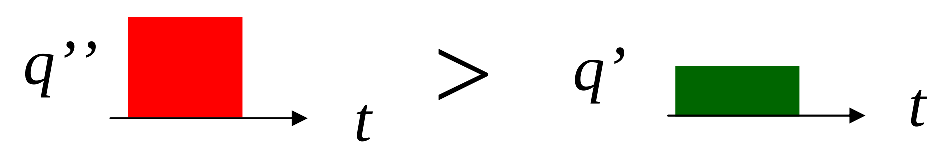
arrival power *q*''

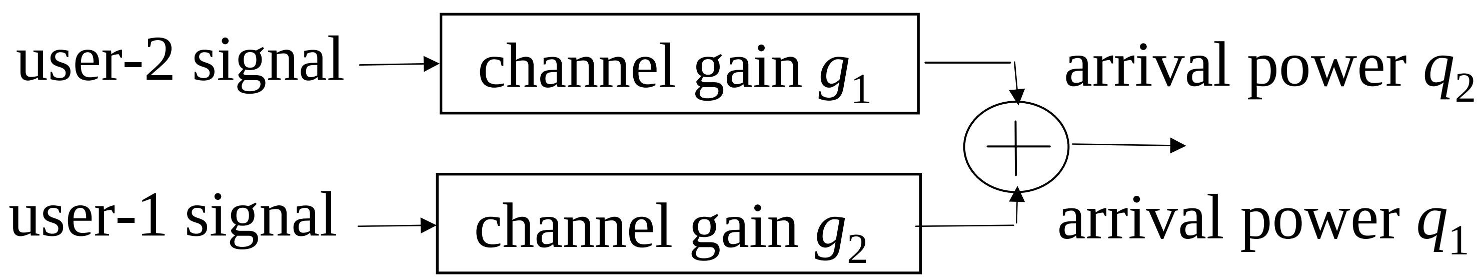
arrival power *q*'



Assume that *q*'' > *q*'.

Power Matching Strategy

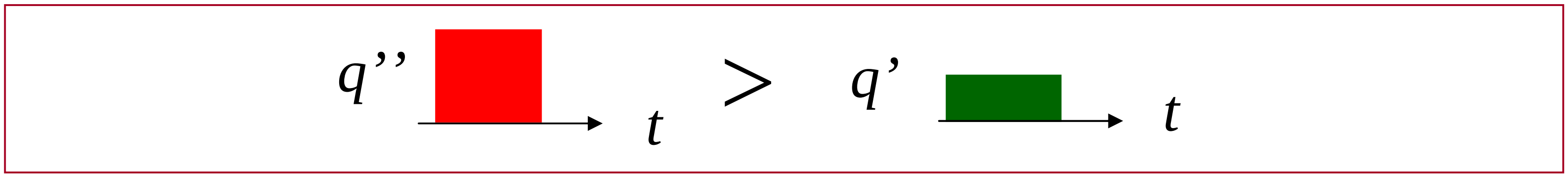


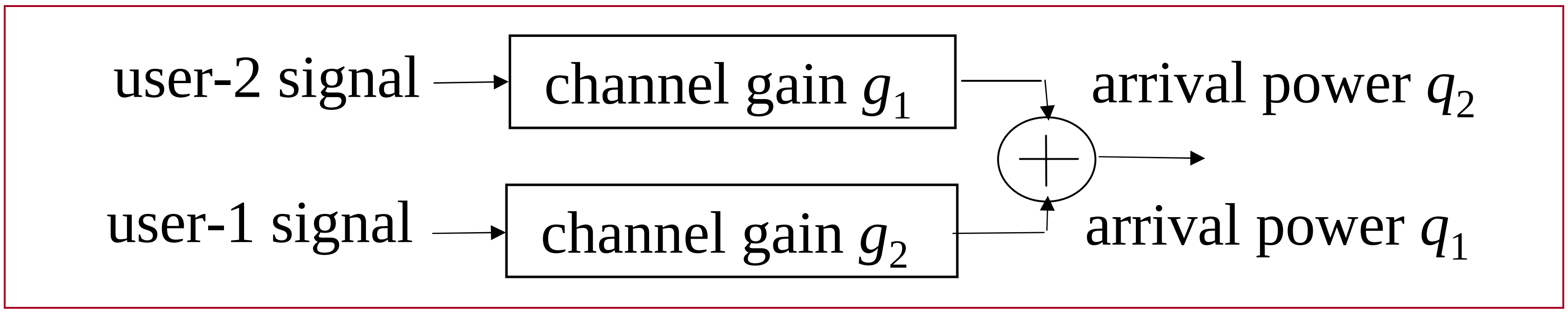


The question is to assign (q', q'') to (q_1, q_2). If we do it randomly, the resultant sum-power is the same as TDMA.

However, we can do better.

Power Matching Strategy





(1) Assign $q_2 = q''$ and $q_1 = q'$. Then

sum transmitted power = $q''/g_2 + q'/g_1$

(2) Assign $q_2 = q'$ and $q_1 = q''$. Then

sum transmitted power = $q''/g_1 + q'/g_2$

Assume $q'' > q'$. Which method leads to smaller transmitted power?

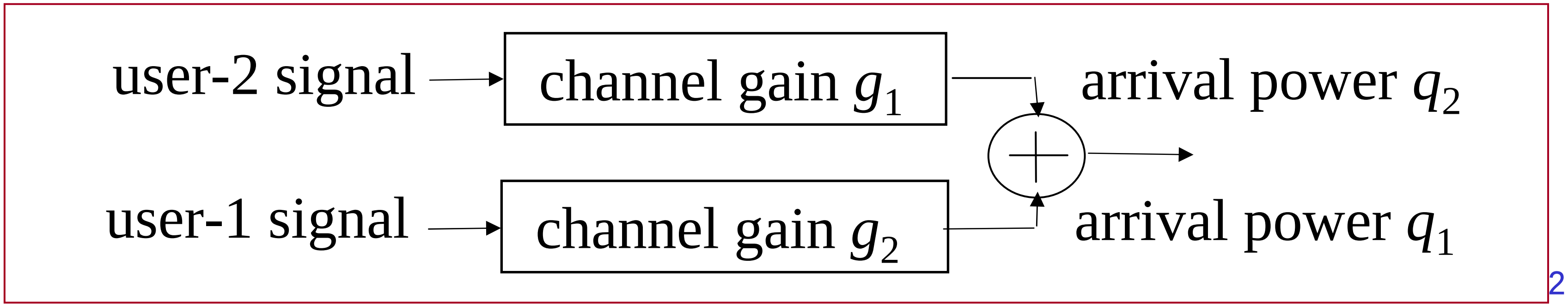
Multi User Gain in SISO Channels

Transmitted power can be minimized by matching smaller receiving power with smaller channel gain.

This principle can be generalized to any number of users.

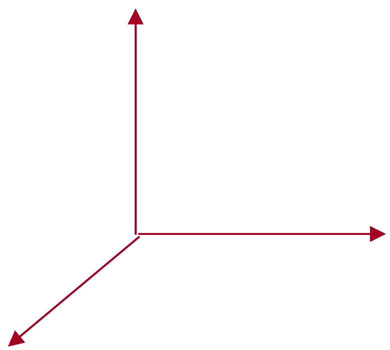
This is the reason of multi-user gain in multi-user SISO fading channels.

There is no such gain in AWGN (or when g_1=g_2).



Multi-User Gain in MIMO Channels

In a MIMO channel, there is one more reason for multi-user gain. A MIMO system provides more dimensions for signaling. This is explained in the following example.

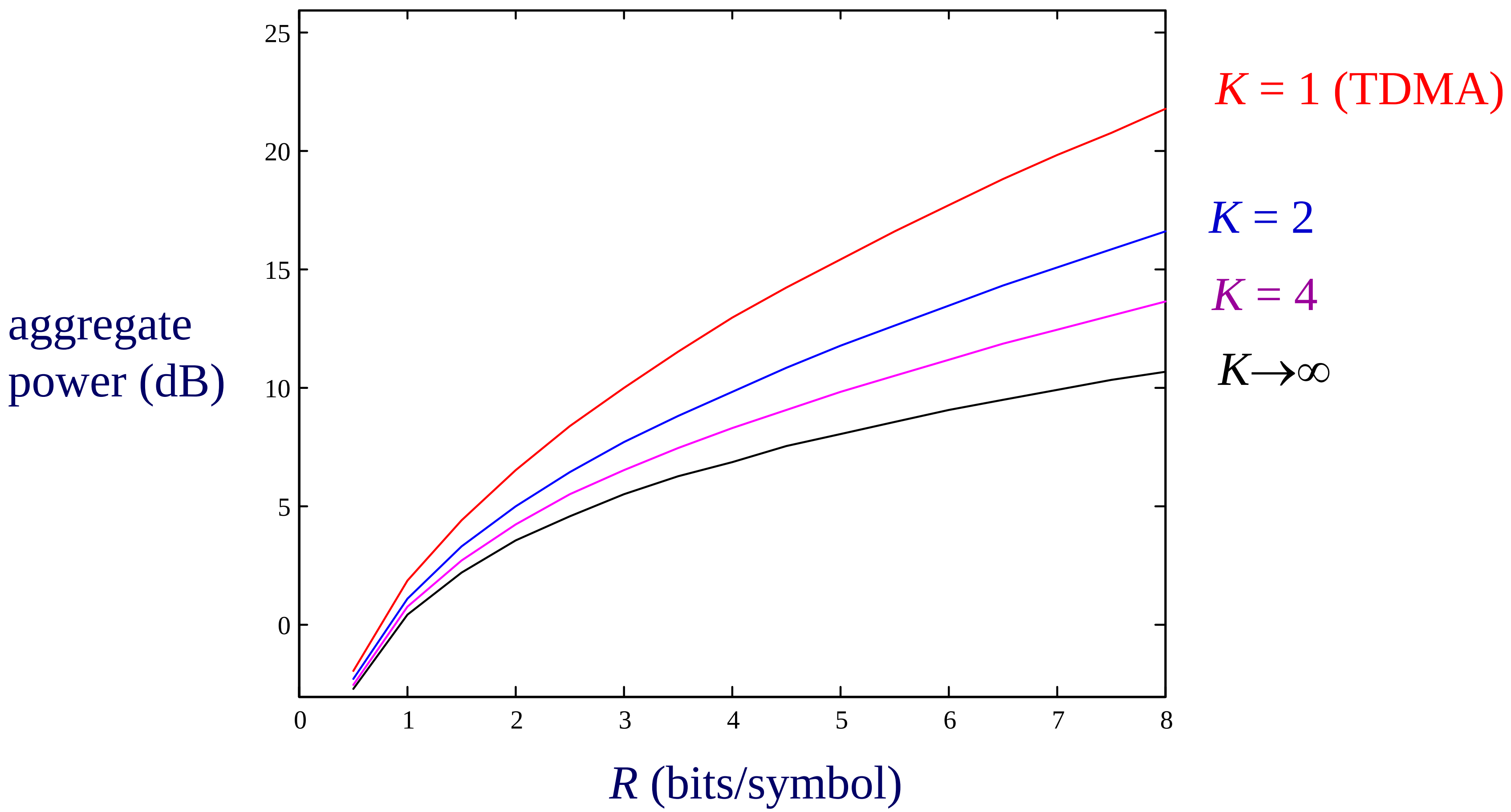


Three receiver antennas can provide a maximum three degrees of freedom. We need the help of the transmitters to achieve this degree of freedom. (We may view each degree of freedom as an eigenvector.)

If there is one user, we need more antennas to achieve these degrees of freedoms.

However, if there are multiple users, we can achieve these degrees of freedoms more easily by allowing con-current transmission.

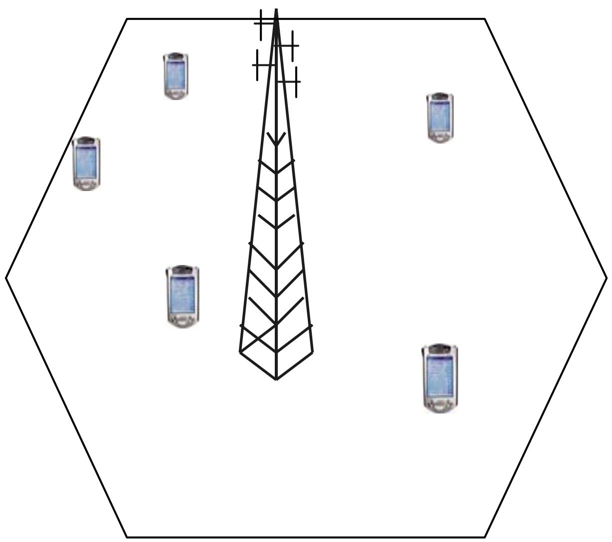
An Example: 2×2 MAC

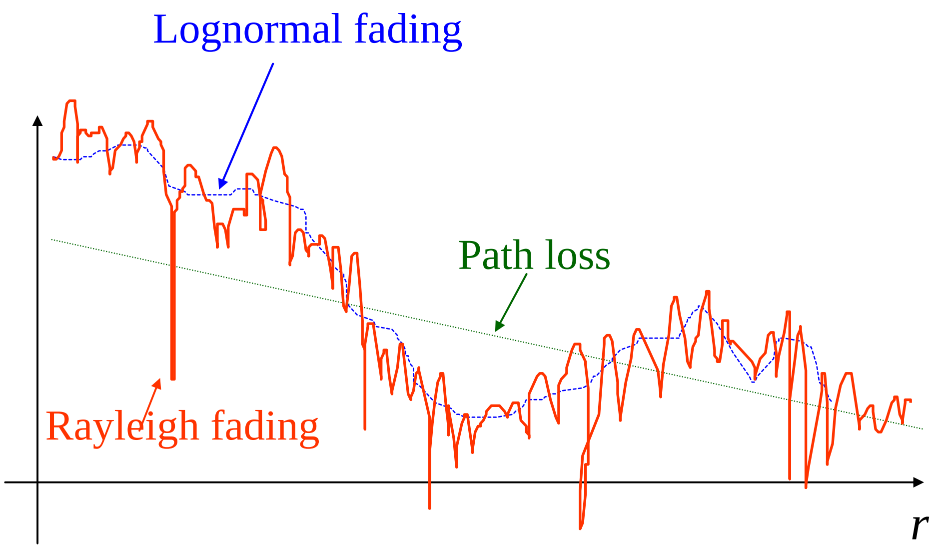


More concurrent transmitting user, more gain.

Assumptions

- Delay sensitive services without scheduling.
- Perfect channel state information (CSI).
- Channel includes path loss, lognormal and Rayleigh fading





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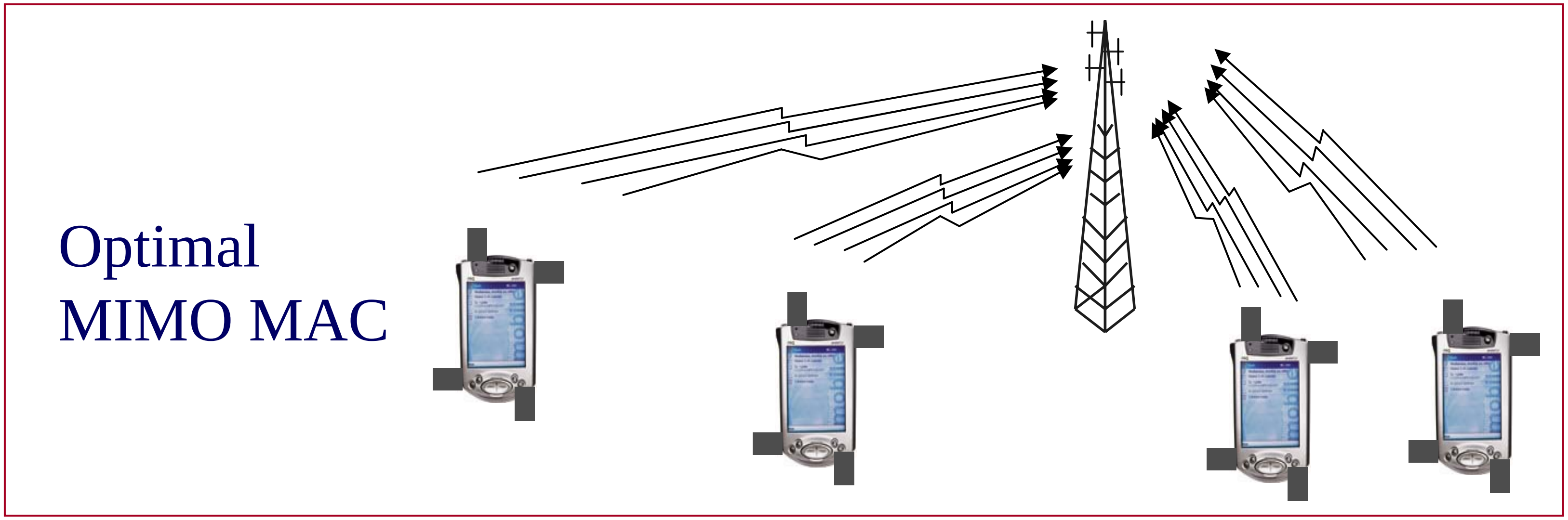
The MEB Strategy

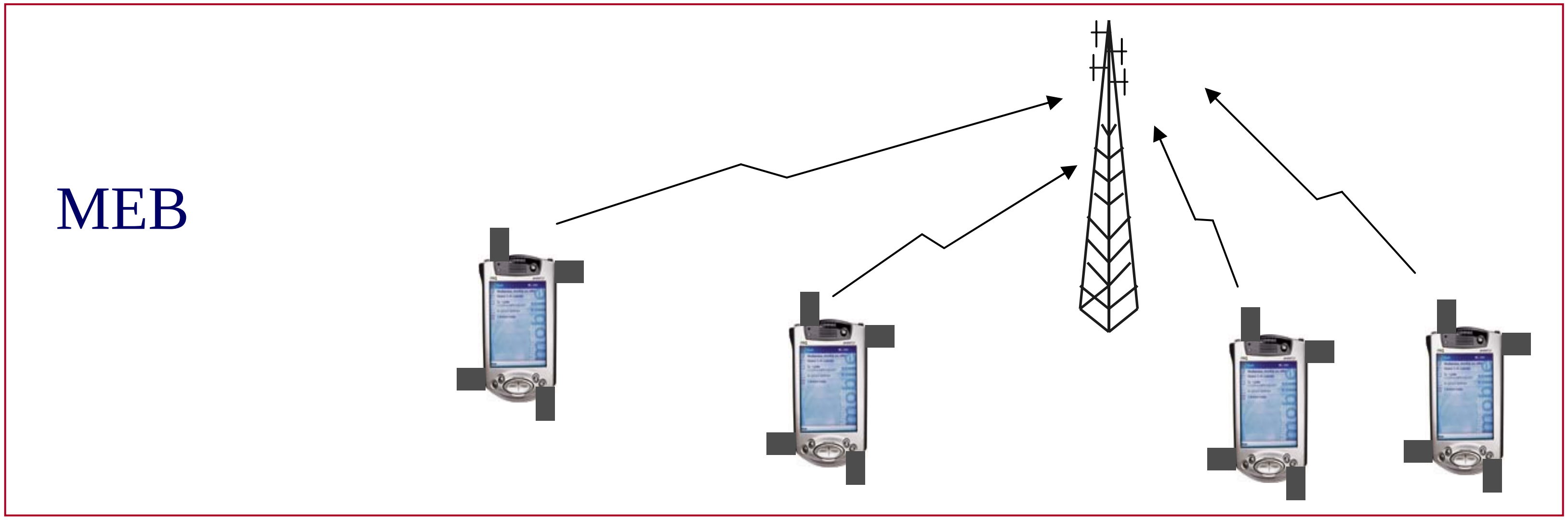
- Each mobile transmits only on its maximum eigenmode.
- Power allocation is applied according to the maximum eigenmode gains $\{g_k\}$.
- Stripping decoding is applied at the receiver. The user with the largest channel gain is decoded first.

The Advantages of MEB

simple and nearly optimal.

Optimal MIMO MAC and MEB





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The 1×1 Single-User Solution

Let $|h_k|^2$ be the channel gain for user k. From Shannon formula

$$R = \log_2\left(1 + \frac{|h_1|^2 p_1}{N_0}\right)$$

The power is

$$p_1 = N_0(2^R - 1)\cdot\frac{1}{|h_1|^2}$$

The Average power is

$$E(p_1) = N_0(2^R - 1)\int_0^{\infty}\frac{1}{g} f(g)\,dg$$

Where $f(x)$ is the pdf of $|h_1|^2$.

MEB in a 1×1 SISO Channel

- For SISO, MEB is capacity achieving.
  (There is only one eigenmode for each user.)
- In this case, the minimum sum-power (MSP) can be easily computed. An explicit expression is available for $K \to \infty$.

For detail, see

Peng Wang, Jun Xiao, and Li Ping, "Comparison of orthogonal and non-orthogonal approaches to future wireless cellular systems," *IEEE Vehicular Technology Magazine*, vol. 1, no. 3, pp. 4-11, Sept. 2006.

The 1×1 SISO Solution

Let K be the number of users number;

$|h_k|^2$ be the channel gain for user k;

$F_{1\times1}(\cdot)$ be the CDF of $|h_k|^2$ in a 1×1 channel.

The minimum sum-power (MSP) is

$$MSP_{1\times1}(K,R) = K\cdot(1-2^{-R/K})\cdot\sum_{k=1}^{K}\frac{2^{Rk/K}}{|h_k|^2}\cdot\frac{1}{K}$$

$$\approx K\cdot(1-2^{-R/K})\cdot\sum_{k=1}^{K}\frac{2^{Rk/K}}{F_{1\times1}^{-1}(k/K)}\cdot\frac{1}{K}$$

When $K\to\infty$,

$$MSP(K,R) = N_0 R\ln 2\int_0^\infty \frac{2^{CF(g)}}{g} f(g)dg$$

MIMO MAC Minimum Sum Power

- Now consider the MSP for an $N \times M$ MIMO MAC.
- The derivation will be based on upper and lower bounds.
- When $K \to \infty$, the upper and lower bounds converge so we obtain the true MSP.
- The upper bound is derived from a realizable system using maximum eigenmode beamforming (MEB).
- The lower bound is derived from an idealized parallel channel.

Upper Bound (Achievability)

- The MEB provides an upper bounds on the MIMO MSP.

$$MSP_{N\times M}(K,R) \le R\ln 2 \sum_{k=\varepsilon K}^{K} \frac{(1+\delta)2^{R(k-1)/MK}}{F_{N\times M}^{-1}(k/K)} \cdot \frac{1}{K}$$

where δ is an arbitrarily small constant, ε the outage probability and $F_{N\times M}(\cdot)$ the CDF of the maximum singular value.

- When $K \to \infty$,

$$MSP_{N\times M}(K,R) \le R\ln 2 \int_0^1 \frac{2^{Rt/M}}{F_{N\times M}^{-1}(t)} dt$$

Proof

$$SNR_k = \frac{p_k d_{k,\max}^2 \, | \boldsymbol{u}_{k,\max}^H \boldsymbol{u}_{k,\max} |^2}{1 + \sum_{i=1}^{k-1} p_i d_{i,\max}^2 \, | \boldsymbol{u}_{k,\max}^H \boldsymbol{u}_{i,\max} |^2} = \frac{p_k d_{k,\max}^2}{1 + \sum_{i=1}^{k-1} p_i d_{i,\max}^2 \phi_{\boldsymbol{k,i}}}$$

$$\lim_{K \to \infty} SNR_k = \frac{\left(2^{R/K} - 1\right)\left(1 + (2^{R/K} - 1)/M\right)^{k-1}(1+\delta)}{1 + \sum_{i=1}^{k-1}\left(2^{R/K} - 1\right)\left(1 + (2^{R/K} - 1)/M\right)^{i-1}(1+\delta)\frac{1}{M}}$$

$$\geq \frac{\left(2^{R/K} - 1\right)\left(1 + (2^{R/K} - 1)/M\right)^{k-1}}{1 + \frac{2^{R/K} - 1}{M}\sum_{\boldsymbol{i}=1}^{k-1}\left(1 + (2^{R/K} - 1)/M\right)^{i-1}}$$

$$= 2^{R/K} - 1$$

$$\lim_{K\to\infty}\sum_{k=\varepsilon K}^{K} p_k = \lim_{K\to\infty}\sum_{k=\varepsilon K}^{K} \frac{(1+\delta)\, 2^{R(k-1)/MK} R \ln 2}{F_{N\times M}^{-1}(k/K)} \cdot \frac{1}{K} = (1+\delta) R \ln 2 \cdot \int_{\varepsilon}^{1} \frac{2^{Rt/M}}{F_{N\times M}^{-1}(t)} dt$$

Lower Bound

MIMO: $$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{n}$$

Its minimum sum-power (MSP) is lower bounded by that of the following idealized system:

Parallel Channel: $$\boldsymbol{y} = \sum_{k=1}^{k} d_k^{\max} \boldsymbol{I}_M \boldsymbol{x}_k + \boldsymbol{n}$$

where $d_k^{\max}$ is the maximum singular value of $\boldsymbol{H}_k$. Each user in this channel sees M parallel sub-channels. Thus

$$MSP_{N\times M}(K,R) \geq R \ln 2 \int_0^1 \frac{2^{Rt/M}}{F_{N\times M}^{-1}(t)} dt$$

same as the upper bound when $K \to \infty$

Asymptotic MIMO MSP

- When $K \to \infty$, the upper and lower bounds converge so we have

$$\lim_{K\to\infty} MSP_{N\times M}(K,R) = R\ln 2\int_0^1 \frac{2^{Rt/M}}{F_{N\times M}^{-1}(t)}dt$$

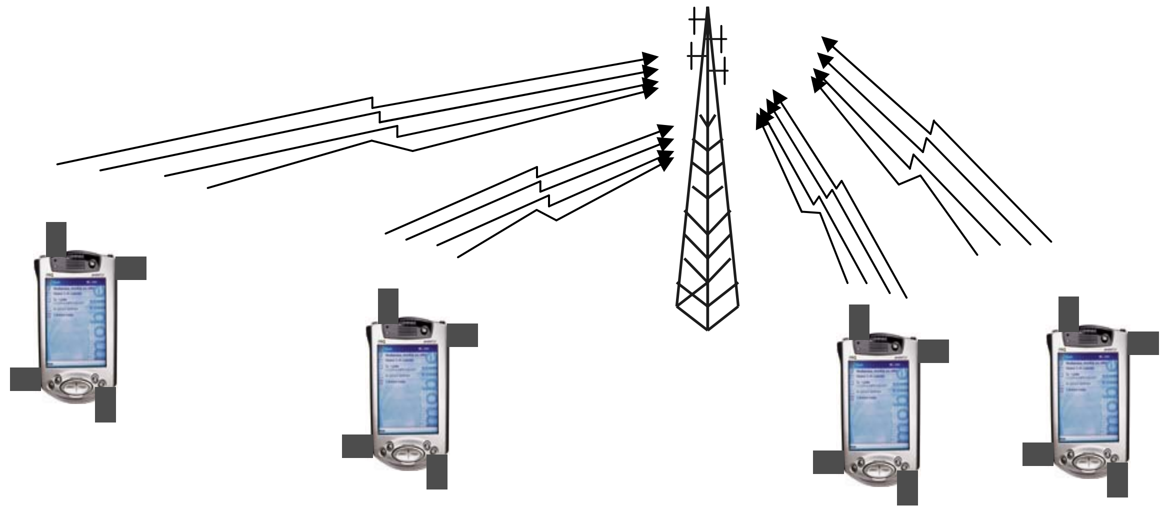
- So, MEB is asymptotically optimal when $K \to \infty$.
- MEB is nearly optimal even for a quite small K.

For detail, see

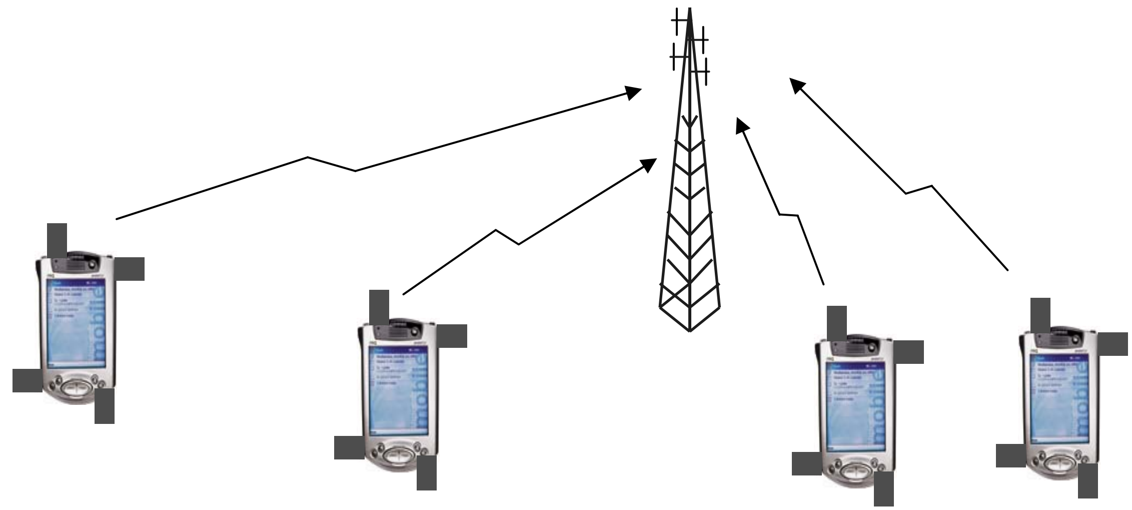
Li Ping and Peng Wang, "Multi-user gain and maximum eigenmode beamforming for MIMO systems with rate constraints," *IEEE Inform. Theory Workshop (ITW)*, Bergen, Norway, July 1-6, 2007.

Optimal MIMO MAC and MEB

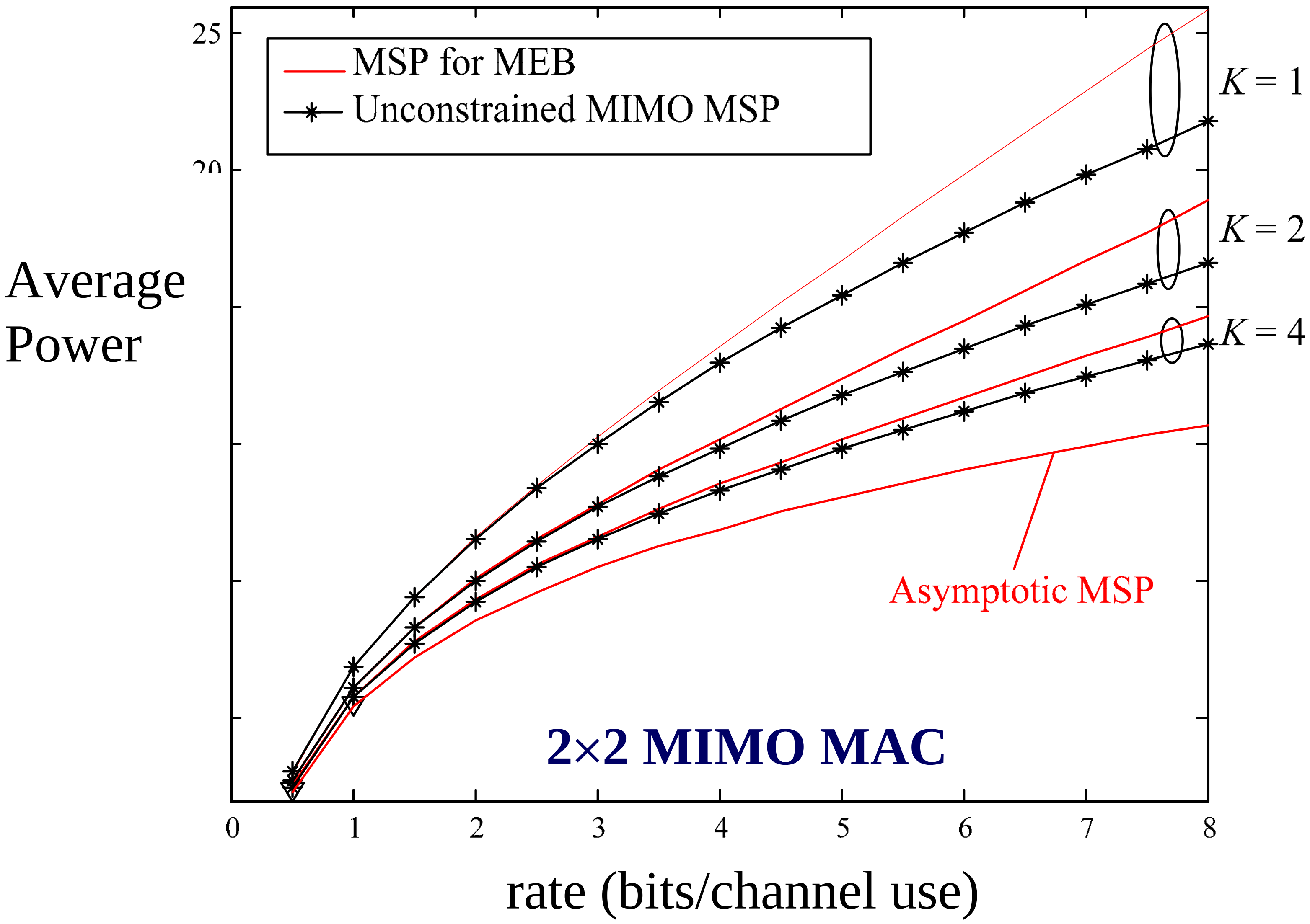
Optimal
MIMO MAC



MEB



MEB Is Nearly Optimal



Some Intuitions

SISO asymptotic MSP

$$MSP_{1\times 1}(K,R) = R\ln 2\int_0^1 \frac{2^{Rt}}{F_{1\times 1}^{-1}(t)}dt$$

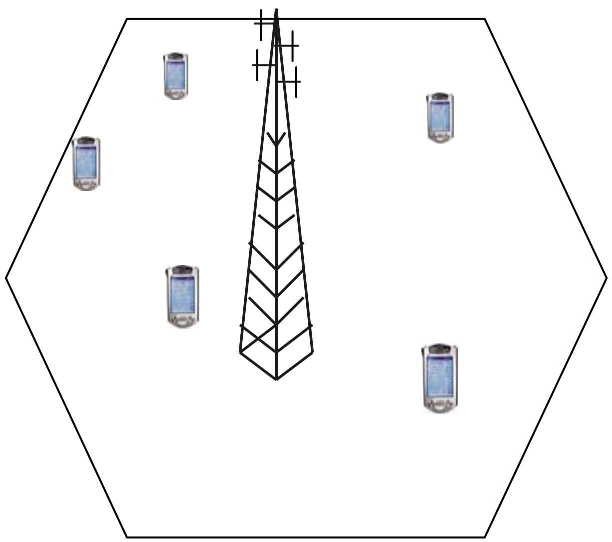
MIMO asymptotic MSP

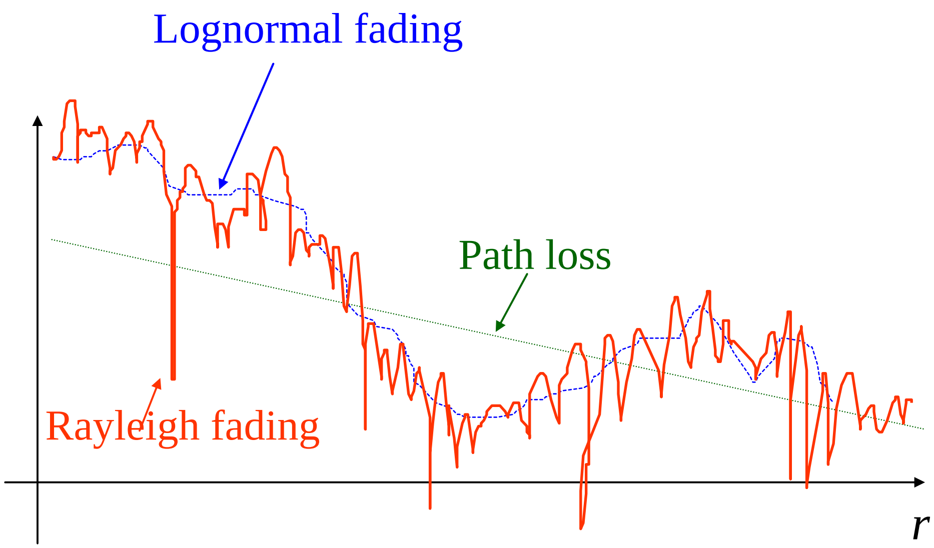
$$MSP_{N\times M}(K,R) = R\ln 2\int_0^1 \frac{2^{Rt/M}}{F_{N\times M}^{-1}(t)}dt$$

- The degrees of freedom is increased by M times.
- Why M? Effectively, we have a $KN\times M$ system. The limiting factor is M.
- What is the impact of N? $F_{N\times M}(\cdot)$

Conditions

- Delay sensitive services without scheduling.
- Perfect channel state information (CSI).
- Channel includes path loss, lognormal and Rayleigh fading





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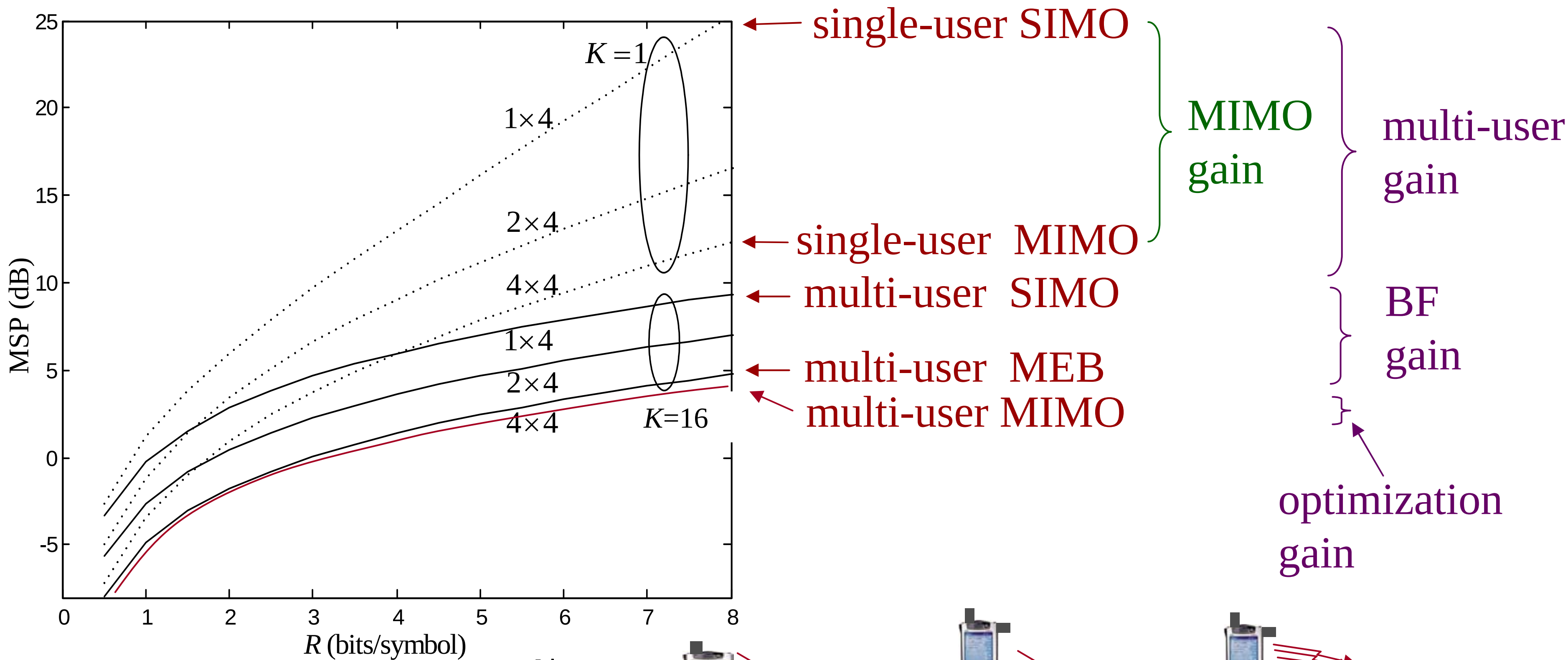
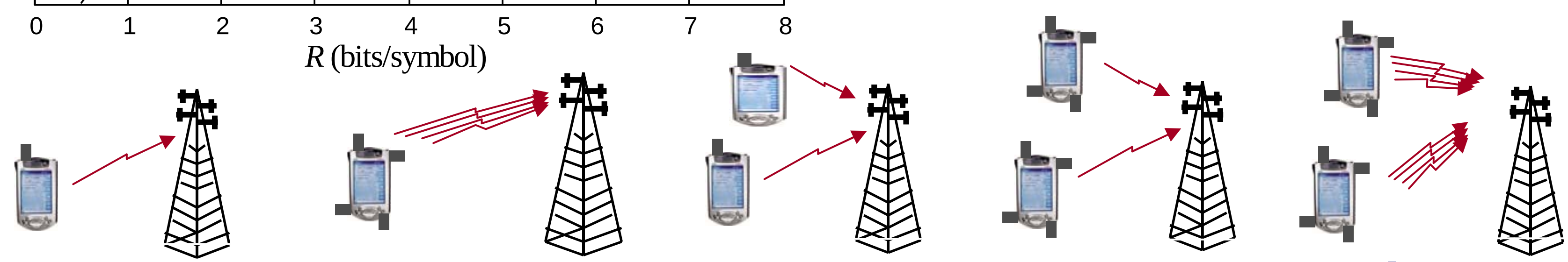
Comparison of Different Gains





single-user SIMO

Single-user MIMO

multi-user SIMO

Multi-user MEB

multi-user MIMO

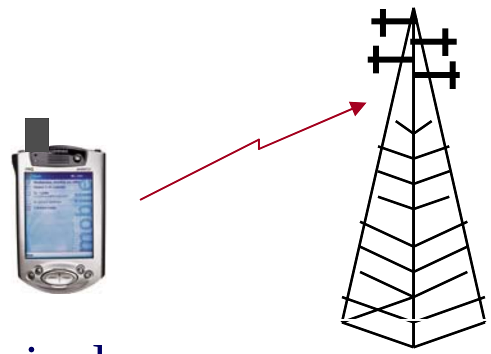
Some details …

For detail, see

Peng Wang and Li Ping, "On multi-user gain in MIMO systems with rate constraints," *IEEE GlobeCom*, Washington, DC, USA, Nov. 26-30, 2007.

(1) Single-User SIMO

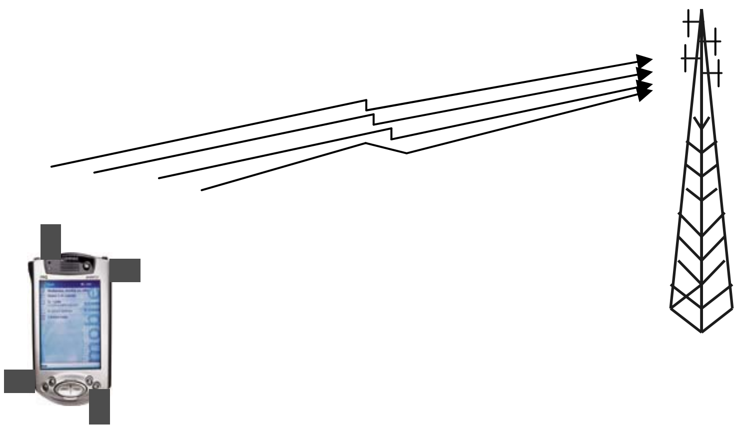
This is used as the reference.



single-user
SIMO

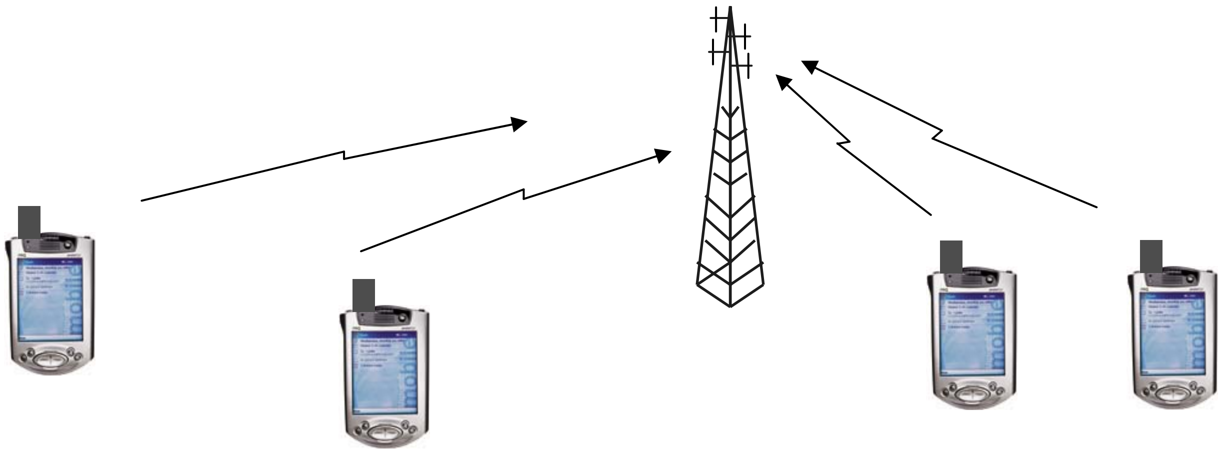
(2) Single-User MIMO

- SVD is sensitive to channel information error (water-filling).
- Bulky .
- Closely located antennas may be rank-deficient. This is a serious problem for a handset unit.



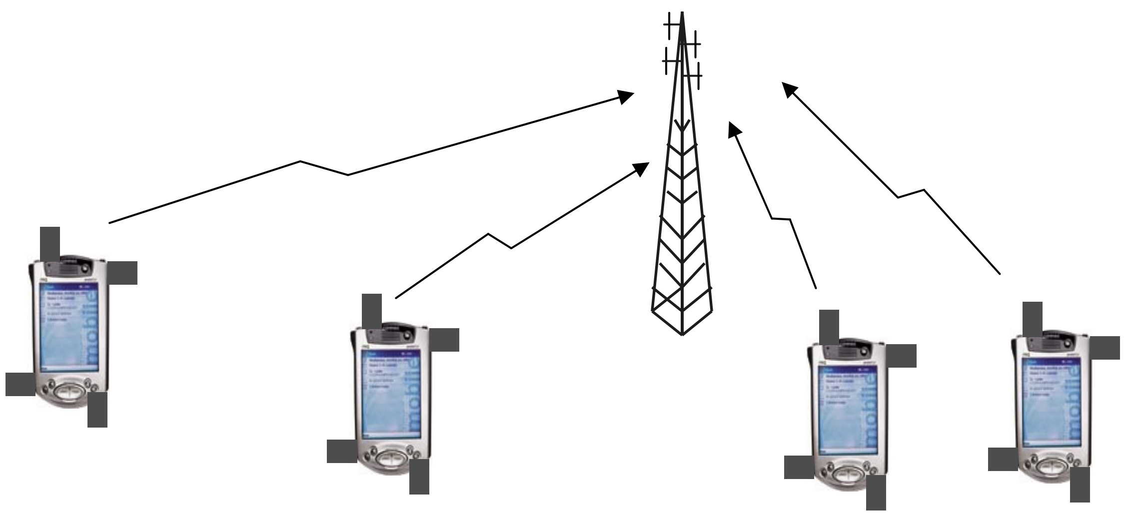
(3) Multi-User SIMO

- SIMO MAC is a special case of MIMO MAC.
- SIMO MAC can also achieve good MUG.
- SIMO MAC requires only one antenna per handset.



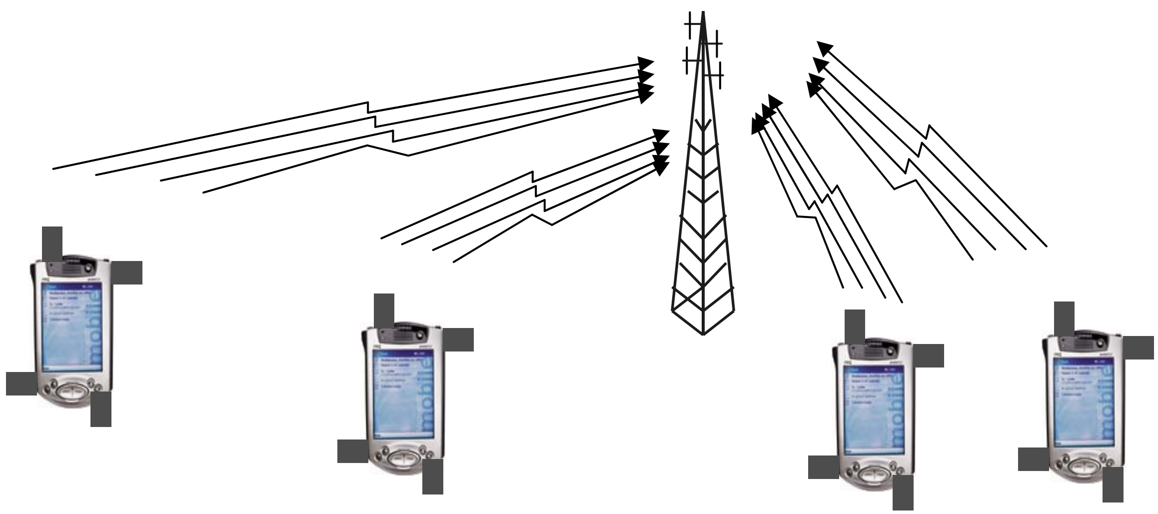
(4) Multi-User MEB

- Easy to design (no water filling).
- Robust against imperfect CSI
- Near optimal performance with multi-user detection.
- But, multiple transmit antennas.
- Rank deficiency is not a problem.

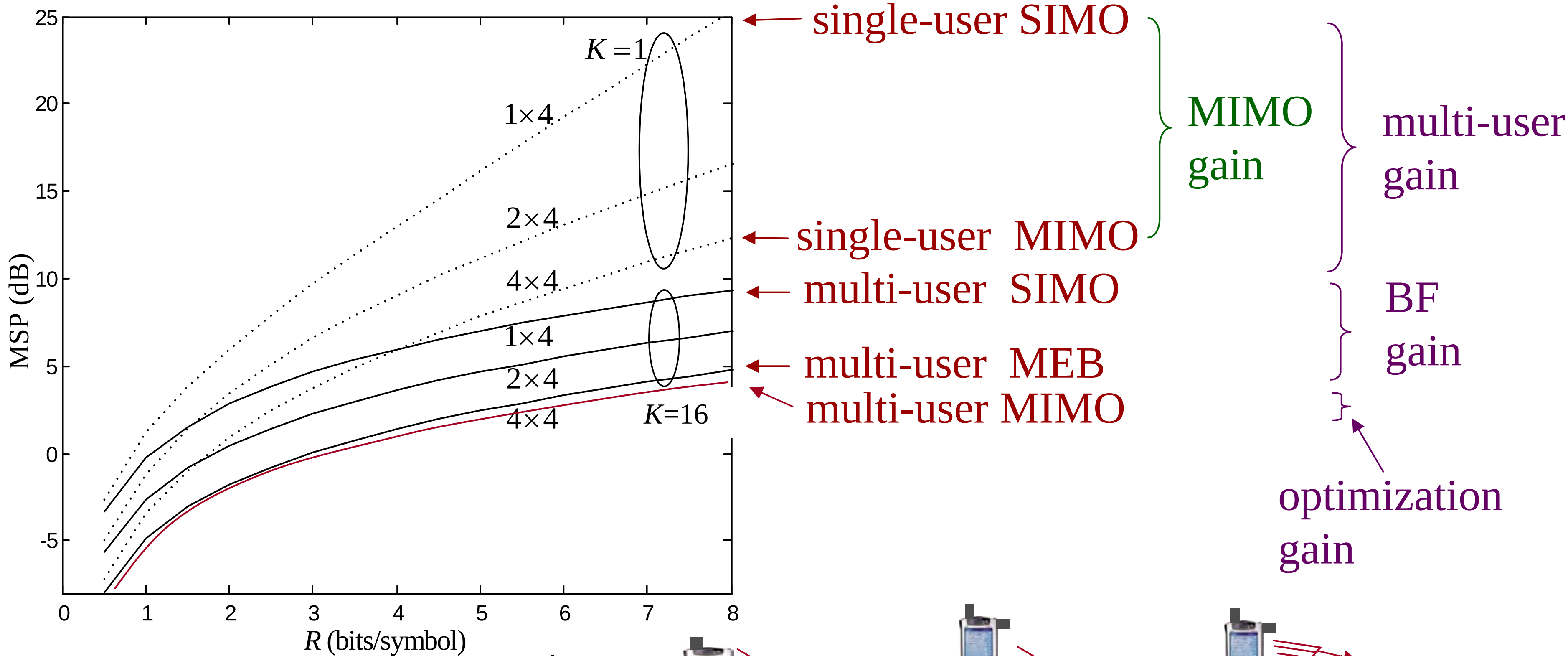


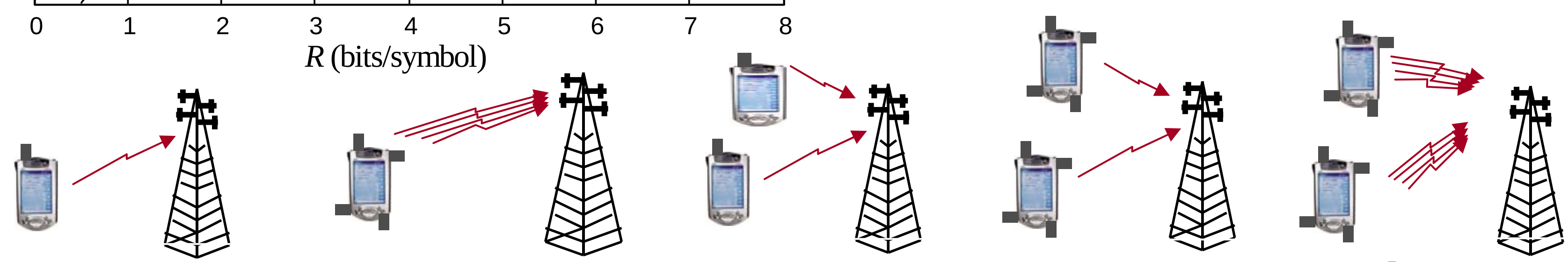
(5) Multi-User MIMO

- Very sensitive to channel information error.
- Bulky .
- Very complicated to optimize.



Comparison of Different Gains





single-user SIMO

Single-user MIMO

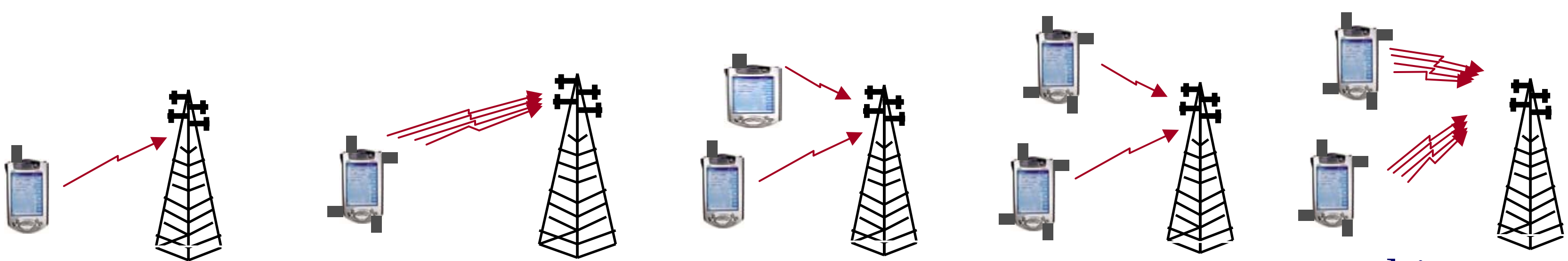
multi-user SIMO

Multi-user MEB

multi-user MIMO

Types of Gains in Multi-User MIMO

- MIMO gain	achievable by single-user MIMO
- Multi-user gain	achievable by multi-user SIMO
- Beamforming gain	achievable by multi-user MEB
- Optimization gain	achievable by multi-user MIMO



single-user SIMO — Single-user MIMO — multi-user SIMO — Multi-user MEB — multi-user MIMO

Optimization Gain

Optimization involves

- decoding order optimization,
- correlation matrix optimization,
- singular value decomposition (SVD),
- eigenmode water filling.

Therefore to achieve optimization gain is a very complicated task.

As we can see, the difference between MEB and optimal MIMO
Is marginal. Thus optimization gain is not significant.

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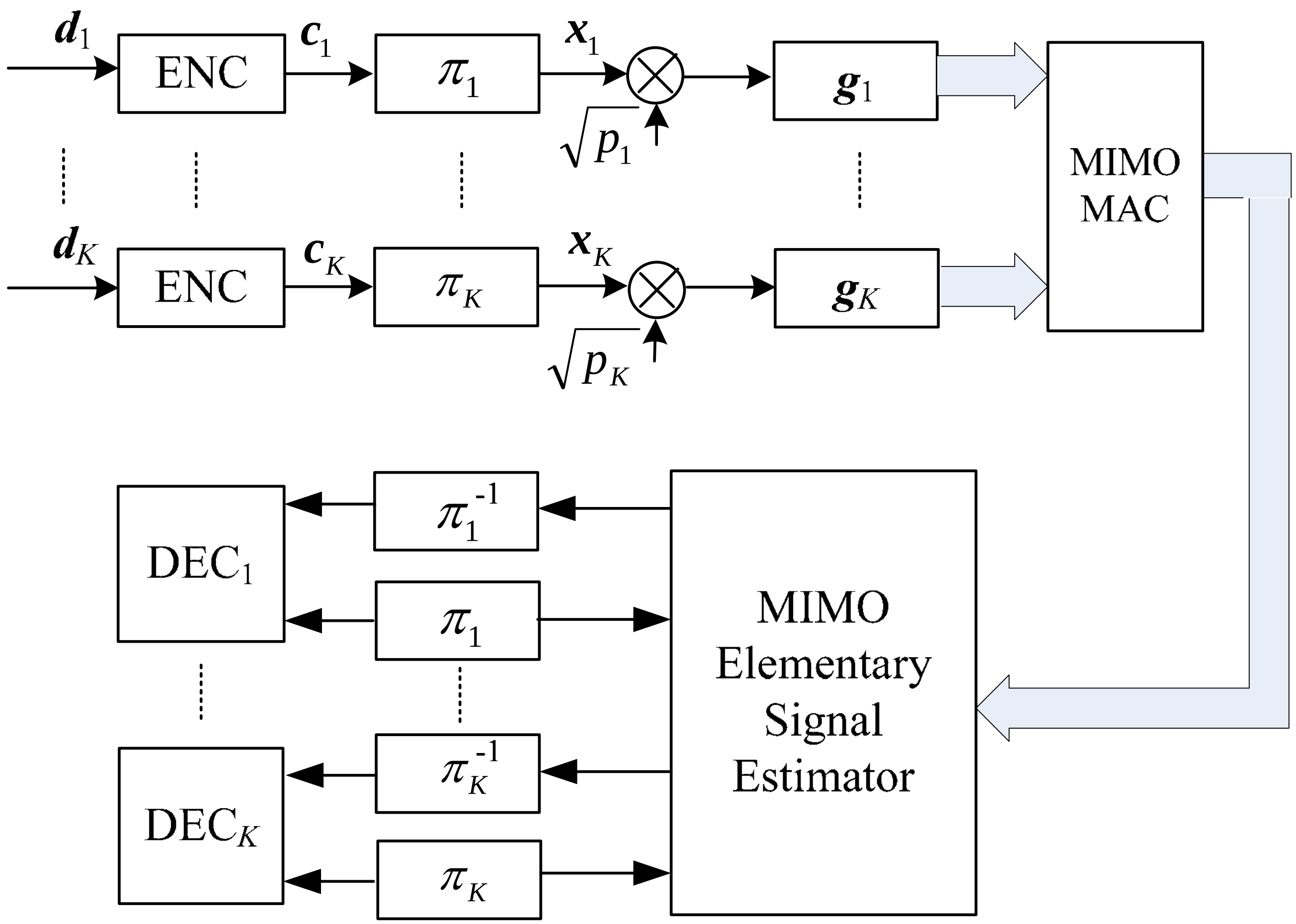
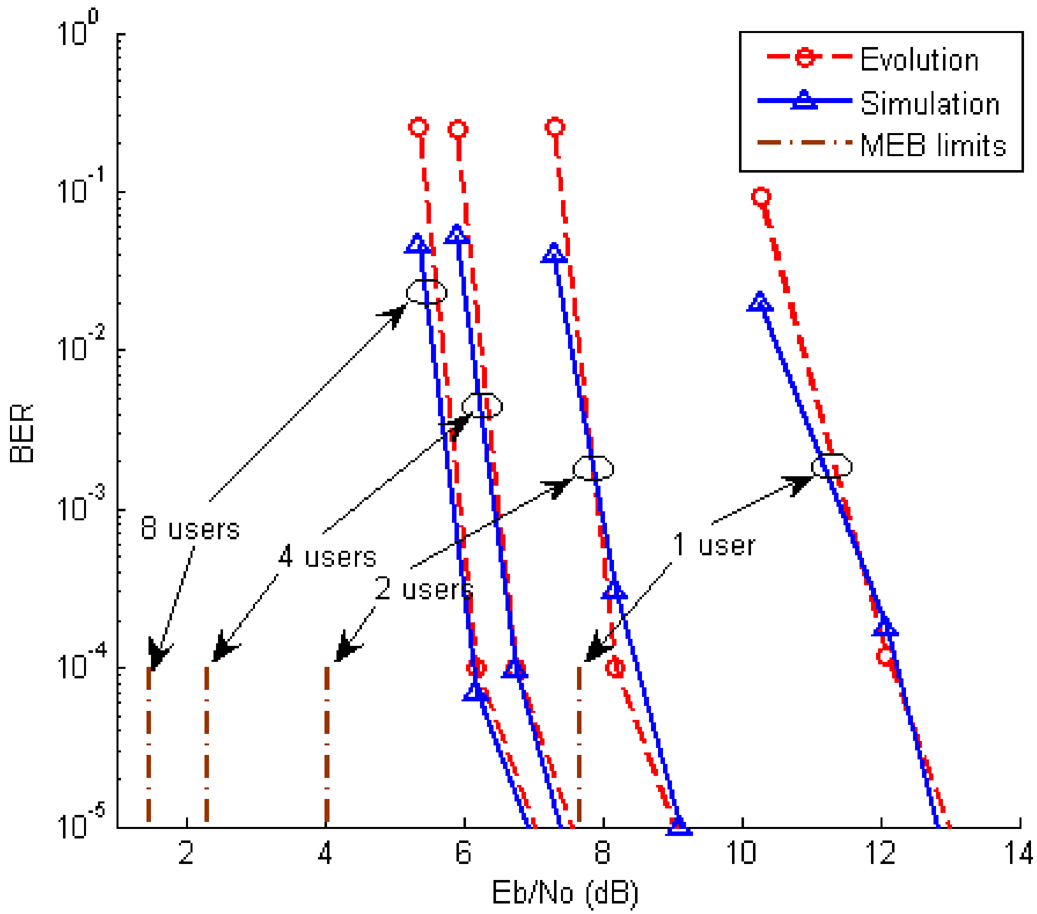
MEB-Based IDMA



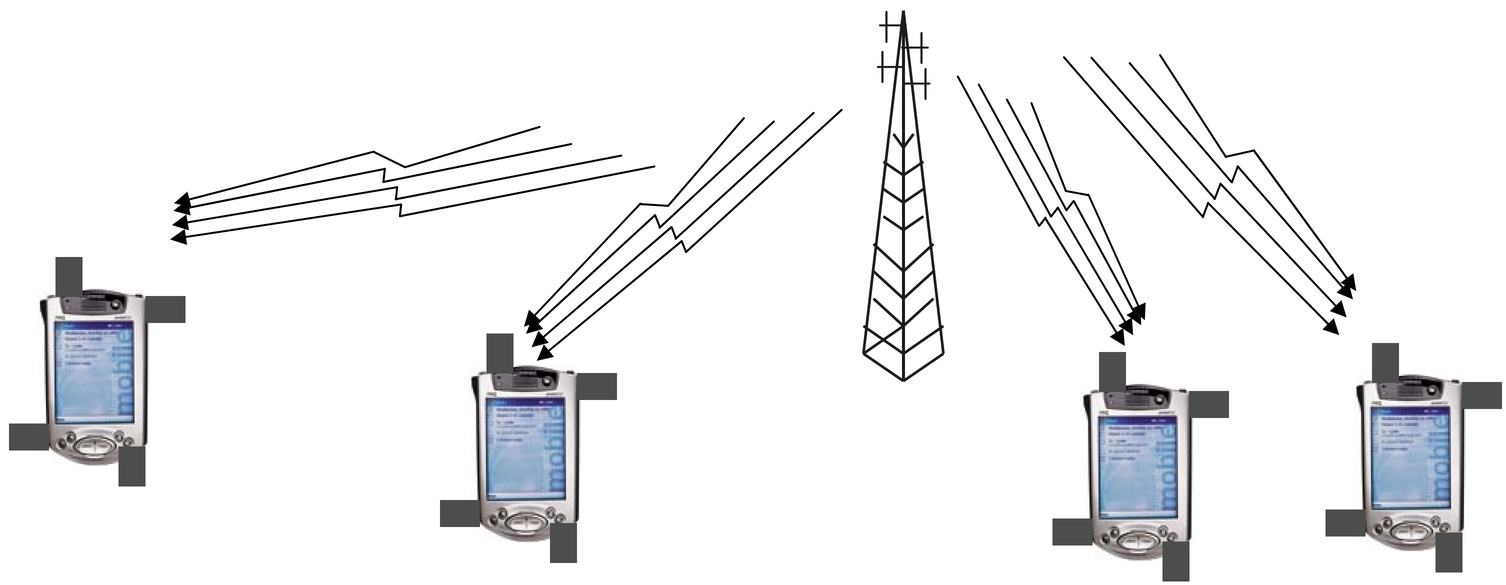
Performance of MEB-Based IDMA

R = 4 bits/symbol



MIMO Broadcasting and Duality

- Based on the MAC-BC duality principle (Yu, Cioffi, Vishwanath, Jindal, Goldsmith), the conclusions in this talk can be also applied to BC channels.
- For simplicity, we will concentrate on MIMO MAC below.



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Conclusions

Optimal MIMO systems are bulky and sensitive. They are also complicated to design and to decode.

Multi-user SIMO can achieve very impressive gain.

If multiple antennas are affordable, MEB is a low-cost, almost optimal strategy.

IDMA provides an efficient framework to realize MUG.

Thank You !