

MARGINAL AND TRACING PRICING OF TRANSMISSION: AN EMPIRICAL COMPARISON

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Abstract – The evolution of the electricity industry into a decentralised structure has highlighted the pivotal role of transmission pricing. In a first-best situation, deriving transmission costs should be based on the marginal pricing of transmission losses. The advantage of this method in guaranteeing economic efficiency is diminished by the volatility of its charges and the practical difficulties surrounding its implementation in a deregulated market. Alternative methods such as the ‘Postage Stamp’ method are used due mainly to its practicability. A new method has been lately presented which is based on the notional assumption of tracing electricity on the transmission network. This paper aims at exploring these issues with special attention on the comparison between the marginal and tracing charging for transmission. This comparison is based on real data for an actual network. The main result of the paper is that there is a high similarity between the marginal and tracing charges. This is especially true for zonal charges in a network which is spread over large geographical areas. In addition to the minimal loss in economic efficiency, the tracing charges are transparent and easy to implement.

Keywords: *Transmission pricing, deregulation*

1 INTRODUCTION

THE uncertainty introduced by the unbundling of the industry and the rapid development of competitive bulk power markets has increased the importance of transmission pricing. Due to the conflicting objectives of any transmission pricing scheme, the methods proposed, so far, to deal with this issue have ranged from the practical and simple to the ideal and complex. One of the latter methods is that of marginal cost pricing [1], which, despite its advantage of attaining economic efficiency, is non-transparent and may be difficult to implement in a large interconnected network. This paper aims at presenting the charges using this method and comparing them with those of the novel method of electricity tracing [2].

2 MARGINAL PRICING OF TRANSMISSION LOSSES

The spatial factor in an electricity system requires that the minimisation of the total cost of production includes the transmission costs between nodes. If there are no transmission costs, the price of electricity at each node is the same and should be equal to the (system) marginal generation cost. Since transmitting electricity is not

costless, the differences in nodal prices reflect the transmission costs and should (in the absence of line congestion) equal to the marginal transmission losses. The optimal price of electricity at node k can be seen as the value of electricity at the reference swing bus, p^* , modified by the cost of transporting it from the swing bus to that node:

$$p_k = p^* \left[1 + \frac{\partial \text{losses}}{\partial d_k} \right] \quad (1)$$

where d_k is the demand at node k . The derivative $\partial \text{losses} / \partial d_k$ is often referred to as the marginal Transmission Loss Factor (TLF) at node k .

In a pool with the true optimal nodal prices, there is no need to decompose the nodal price into generation and losses as the transmission price is contained in the overall nodal price p_k . However, in order to facilitate transaction-based trading in the system, often loss charges are calculated and charged separately from the main energy auction. A bilateral transaction from node i to node j would then pay the loss charge equal to:

$$(\text{loss charge})_{i,j} = p^* \left[\frac{\partial \text{losses}}{\partial d_j} - \frac{\partial \text{losses}}{\partial d_i} \right] \quad (2)$$

Marginal pricing for losses sends the optimal pricing signals to all the users but its implementation can be very difficult. Firstly, calculation of marginal loss factors is cumbersome, especially for a large interconnected system. Secondly, the methodology itself is non-transparent and this non-transparency increases with the system size. Thirdly, the marginal loss factors are known to be volatile hence requiring some additional hedging mechanisms. Fourthly, the sum of collected loss charges is not equal to the actual sum of losses (the so-called Merchandise Surplus). Hence an additional mechanism would be necessary in order to distribute the surplus. In the case of transmission pricing of inter-area trades, the methodology would also require that the losses inside each area are also charged on the marginal basis. This however would be very difficult to achieve as each area usually follows their own internal pricing methodology. Making them adopt the same methodology, whether marginal or any other, would probably meet a very strong resistance. In order to overcome all the problems

related to the marginal loss charging the tracing-based methodology has been suggested [2].

3 THE INFLUENCE OF THE MARGINAL GENERATOR

Any additional unit of demand for electricity energy may increase, decrease or have no effect on the net flows along transmission lines. This will, consequently, affect the losses associated with these flows. This impact is linked to the location of the additional source of generation, which depends on the location of the marginal generator. The marginal charges in this paper are obtained from using an AC network model and by utilising actual data from one of the electricity companies in Saudi Arabia. The data include node data for the 16 generators and 109 loads in the transmission network of the company.

The results include the marginal losses for each node when the marginal generator can be any one of the sixteen generators in this network. Thus, these results identify (in term of transmission losses only) for the ranking of generators from the most to the least efficient. In Figure 1, below, three marginal generators (G14, G11 and G4) are selected to show how the marginal losses for each generator in the network vary according to where the marginal generator is located. Changing a marginal generator shifts effectively the loss curve by a constant. This means that although the loss fee charged to a bilateral contract according to equation (2) does not depend on the choice of the marginal generator, the loss charge in a Pool-based nodal market changes the value as well as sign with a different marginal generator. For example, G11 could have positive, zero or negative marginal losses, depending on which generator among the three generators is designated to be the marginal one.

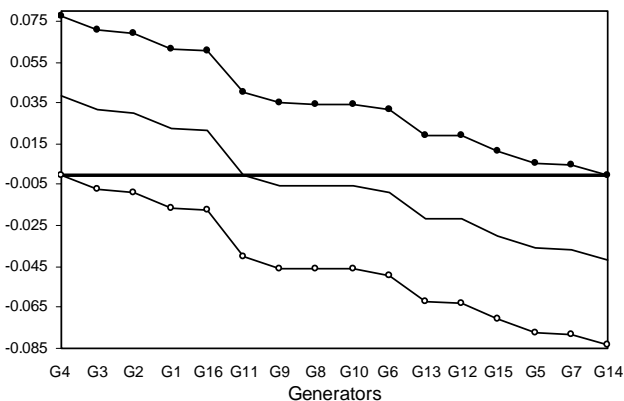


Figure 1: Impact of Marginal Generator Choice on Marginal Losses

Regardless of the marginal generator, G14 always has the lowest marginal losses. This means that it is the most efficient generator in terms of its impact on total losses

and that G4 is the least efficient. The rest of the sixteen generators are ranked in between the two, from the least efficient on the left-hand end of the x-axis to the most efficient on the right-hand end. Being the most efficient in terms of losses is an important factor in economic dispatching where both generation costs and transmission costs are incorporated in the decision of which generator will meet the additional increase in demand. Figure 1, above, illustrates how the choice of the marginal generator clearly influences the marginal losses for each one of the sixteen generators. These fluctuations in marginal losses will naturally result in high volatility in nodal prices.

Obviously when the trading arrangements in electricity markets (pools) are made on the basis of *ex post* prices, there is only one set of prices to consider and there is no room for guess work. The main advantage of these prices is that they are economically efficient, as they should reflect the actual optimal dispatching of the system. However, the price volatility, which results from changing the marginal generator, introduces risk and hedging costs into the process. In addition, trading in electricity as a commodity makes it necessary, especially for wholesale trade, to make use of *ex ante* prices. The parties to the contracts have to agree on the strike price based on certain assumptions about future dispatching conditions.

Consequently, the choice of a generator as the marginal generator makes the contract price predictable but not necessarily unique. The main disadvantage of such prices are that these assumptions can be open to dispute and the prices are less efficient as they are not based on the actual operation of the system. This uncertainty introduced with unbundling could reduce the potential for further trade and might even hinder the process itself. This makes it worthwhile to consider the tracing method which has the advantage of producing charges which are stable and fair. The following sections examine how the tracing (nodal and zonal) charges compare to the ideal as in the marginal charges and how both methods deal with the issue of cost recovery.

4 TRACING PRICING OF TRANSMISSION LOSSES

Conventional wisdom says that it is impossible to trace the flow of power from individual generators to individual loads in meshed transmission networks. Recently however, a novel approach has been developed that challenges this view [2]. Assuming that at any network node the inflows are distributed proportionally between the outflows, it is possible - by following the acyclic directed graph (digraph) of flows in the network - to trace how real and reactive power flows in the network from individual sources to individual sinks. In other words, tracing methodology allows to "tag MWs", i.e. establish physical paths linking generators and the loads.

Consequently transmission charges can be calculated similarly as in the traditional contract path approach. It should be stressed, however, that the tracing paths are based on the physical power flows in the network while the contract paths are based on the financial contracts which usually do not reflect the physics of transmission. The proportionality assumption, which is behind the whole methodology, can be justified using the cooperative game theory [3].

Tracing can be conducted either by a matrix-based [2] or a graph-based algorithm [4]. Although the graph-based approach is intuitively easier to understand, in this paper the matrix version will be presented as it is mathematically more compact and it allows to deal easily with circular flows. Below a short summary will be presented as all the details can be found in [2]. First the loss allocation to the loads (i.e. demand centers) will be discussed followed by the loss allocation to the generators.

Let us assume that all the power flows in a network are known. For the loss allocation to the loads, let us define an unknown *gross branch flow* $P_{i-j}^{(gross)}$ as the sum of the actual real power flow at the sending end of branch $i-j$ plus the sum of all the losses accumulated upstream from branch $i-j$ in the digraph of network flows. Similarly let the *nodal gross power*, $P_i^{(gross)}$, be a total power flow P_i through node i plus the sum of all the losses accumulated upstream from node i . When looking at the nodal inflows:

$$P_i^{(gross)} = \sum_{j \in \alpha_i^{(u)}} |P_{i-j}^{(gross)}| + P_{Gi} \quad (3)$$

where P_{Gi} is the generation at node i and $\alpha_i^{(u)}$ is the set of nodes supplying directly node i (i.e. power must flow towards node i in the relevant lines). Substituting $P_{i-j}^{(gross)} \cong (P_{i-j} / P_j) P_j^{(gross)}$ and noting that $|P_{i-j}^{(gross)}| = |P_{j-i}^{(gross)}|$ gives

$$P_i^{(gross)} - \sum_{j \in \alpha_i^{(u)}} \frac{|P_{j-i}|}{P_j} P_j^{(gross)} = P_{Gi}$$

$$\text{or} \quad \mathbf{A}_u \mathbf{P}_{gross} = \mathbf{P}_G \quad (4)$$

where \mathbf{P}_G and \mathbf{P}_{gross} are the vectors of nodal generations and nodal gross flows, respectively, and \mathbf{A}_u is the *upstream distribution matrix* defined as:

$$[\mathbf{A}_u]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -|P_{j-i}|/P_j & \text{for } j \in \alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

The unknown vector \mathbf{P}_{gross} can be calculated as

$$\mathbf{P}_{gross} = \mathbf{A}_u^{-1} \mathbf{P}_G \quad (6)$$

Now the loss allocated to load i , ΔP_{Di} , can be calculated as the difference between the gross and the actual demand:

$$\Delta P_{Di} = P_{Di}^{(gross)} - P_{Di} = \frac{P_{Di}}{P_i} P_i^{(gross)} - P_{Di} \quad (7)$$

where P_{Di} is the demand at node i .

For the loss allocation to the generators let us define an unknown *net nodal power*, $P_i^{(net)}$, as a total power flow P_i through node i minus the sum of all the losses accumulated downstream from node i in the digraph of network flows. Using a similar reasoning as before, the vector of nodal net flows \mathbf{P}_{net} can be calculated as

$$\mathbf{P}_{net} = \mathbf{A}_d^{-1} \mathbf{P}_D \quad (8)$$

where \mathbf{P}_D is the vector of nodal generations, and \mathbf{A}_d is the *downstream distribution matrix* defined as:

$$[\mathbf{A}_d]_{il} = \begin{cases} 1 & \text{for } i = l \\ -|P_{l-i}|/P_l & \text{for } l \in \alpha_i^{(d)} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where $\alpha_i^{(d)}$ is the set of nodes supplied directly from node i (i.e. power must flow from node i in the relevant lines).

Now the loss allocated to generator i , ΔP_{Gi} , can be calculated as the difference between the actual and net generation:

$$\Delta P_{Gi} = P_{Gi} - P_{Gi}^{(net)} = P_{Gi} - \frac{P_{Gi}}{P_i} P_i^{(net)} \quad (10)$$

In the absence of circular flows the digraph is acyclic and the graph-based and the matrix-based algorithms are computationally equivalent [3]. This can be proved by noting that the matrices \mathbf{A}_u and \mathbf{A}_d are triangular, and the solution of equations (6) and (8) can be obtained by recursive back- and forward substitution.

The proportionality assumption, which was used in

(7) and (10), can be justified using the cooperative game theory [3]. As also shown in [3], tracing methodology can be seen as a compromise between economically optimal marginal pricing and simple averaged (postage stamp) pricing.

5 NODAL CHARGES FOR LOSSES UNDER THE MARGINAL AND TRACING METHODS

The importance of the comparison between the two methods is that it shows if there is any similarity or pattern in the charges of these two methods. The following table shows that there is a relatively strong degree of association between the marginal (M) and tracing (T) charges for losses. This is especially the case when zonal charges are levied on demand centers. On the other hand, postage stamp (PS) charges give very low correlation coefficient, which is almost zero with either marginal or tracing charges.

TABLE 1: CORRELATION COEFFICIENTS FOR LOSS CHARGES

Correlation Coefficients	Nodal		Zonal	
	Ds	Gs	Ds	Gs
r (T,M)	0.7512	0.7684	0.8258	0.61609
r (T,PS)	-6.20E-17	-4.90E-17	2.08E-17	4.95E-17
r (M,PS)	-4.50E-16	1.77E-16	-4.65E-16	2.37E-16

Figures 2 and 3, below, show how losses are allocated to individual generators and demand centres, respectively, according to the marginal and the tracing methods.

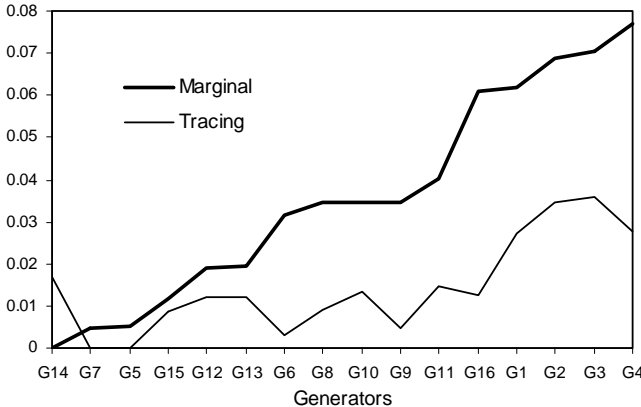


Figure 2: Marginal and Tracing Charges for Generators (Nodal)

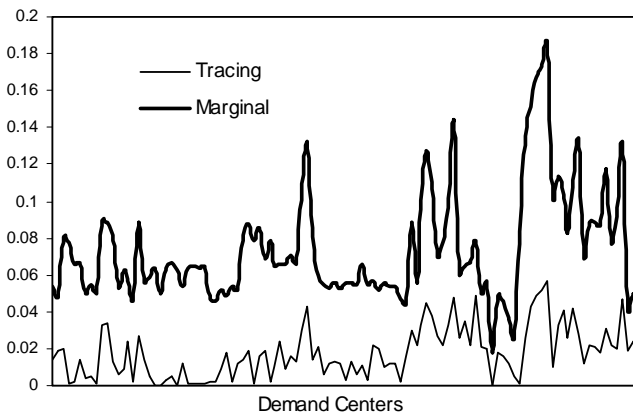


Figure 3: Marginal and Tracing Charges for Demand Centers (Nodal)

Figures 2 and 3 show that the two methods have a similar pattern, but not the same charges. The value and the sign of the marginal losses vary with the choice of the marginal generator while, the allocation of the traced losses is independent of this choice (i.e. only one set of charges). So, the comparison between the two methods should focus on the pattern (which reflects the ranking) of the charges rather than on their respective values. This similarity can be quantified by considering the values of correlation coefficients between the marginal and tracing charges which are 0.77 and 0.75 for generators and demand centres, respectively (also see Table 1).

One observation from Figure 2 is that the differences in charges between the most efficient and the least efficient generator are larger using the marginal method, which shows the advantage of this method in giving a stronger spatial message. However, this large differential in charges would make the marginal method more difficult to implement as some of the distant generation and distribution companies may find it disadvantageous and, hence, unacceptable.

Figure 3 gives more confirmation of the similarity in pattern between the two methods. It shows that the marginal and the traced losses are very similar for those demand centres with very high and those with very low losses. Also, it shows that the demand centres with very low charges according to the marginal method have been given even lower charges according to the tracing method. The possible explanation is that the tracing method has a very strong local effect, where demand centres (generators) which are located near generators (demand centres) would have too low charges.

In addition, Figure 3 shows that the dissimilarity between the two methods is for those demand centres with moderate charges according to the marginal method. This means that the marginal method is able to reflect the small geographical differences between nodes, while the tracing method needs a larger distance between the nodes to give a significant difference in charges.

6 ZONAL CHARGES FOR LOSSES UNDER THE MARGINAL AND TRACING METHODS

The nodal charges, above, indicate that the tracing method gives a strong spatial message when the network is spread over a relatively large geographical area and it has relatively low load density. When there is high load density, it is expected that many demand centres and generators will be relatively close to each other within the same area. In such a case, it is more fitting that the marginal rather than the tracing method is used for the

purpose of nodal pricing. However, in practice zonal charges have more relevance when the network is relatively large.

Figures 4 and 5, below, show the comparison of the two methods as the nodes are grouped into zones. These figures show the similarity between the two methods, especially for the generators located at the extreme ends of the network. For example, the distant generators (G1, G2, G3, and G4) are grouped under zone Z1, and generators G5 and G7 under zone Z2. The two methods, more or less, give different charges for the other zones and this is a confirmation of the ability of the marginal method to reflect small distances between nodes or zones. This grouping has also been done for the other generators (as well as for the demand centres). The comparison of the two methods on the basis of zonal charges gives further similar charges, which more so in the case of demand centres than that of generators.

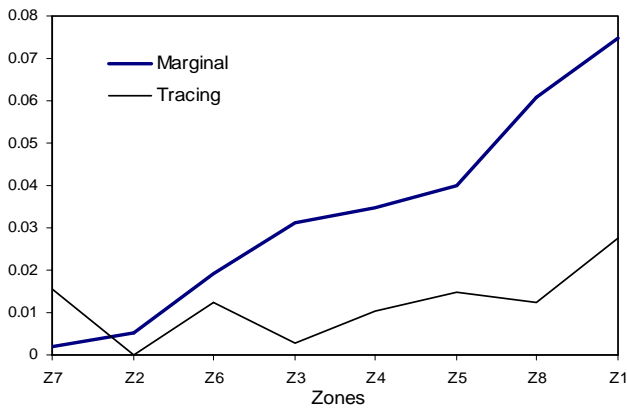


Figure 4: Marginal and Tracing Charges for Generators (Zonal)

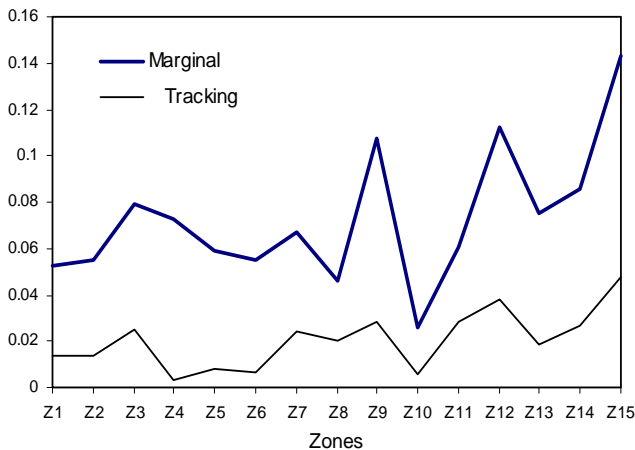


Figure 5: Marginal and Tracing Charges for Demand Centers (Zonal)

Figures 4 and 5 show zonal charges and illustrate that the tracing charges are even more similar to the marginal charges for zonal charges (for demand centres). This similarity can be quantified by considering the correlation coefficients between the marginal and tracing

charges which are 0.62 and 0.83 for generators and demand centres, respectively. This low coefficient for generators is due to the fact that a few generators are located mostly in one zone. Thus, this result not only highlights the limitations of the data used (i.e. just 16 generators), but also confirm the local effect associated with the tracing charges. Therefore, it can be said that the usefulness of the tracing method is greater for zonal rather than for nodal pricing, especially for interconnected networks over vast geographical areas.

7 THE RECOVERY OF TRANSMISSION COSTS

In economic theory, the concept of natural monopoly refers to an activity where the technological advantages of large-scale production preclude efficient competition among smaller companies. This is very much the case with the transmission segment of electricity system where investments are usually characterised as being specific and indivisible (lumpy). Thus, a large proportion of total transmission costs is considered sunk. In this case, the marginal cost price fails to raise sufficient revenues to keep the company operational in the long run. The challenge, therefore, is how to recover the total costs with the least distortion to economic efficiency. Thus, this is a crucial objective that any transmission pricing scheme should satisfy because revenue adequacy is a necessary condition for economic efficiency. The transmission network owners are not expected to invest in additional capacity if their incurred costs are not guaranteed to be recovered. Hence, the choice of transmission pricing scheme must strike a delicate balance between this objective and the other equally desirable objectives.

TABLE 2: THE RECOVERY OF TRANSMISSION LOSSES

Gs	Output (MW)	Marginal Charges in		Tracing Charges in	
		(pu)	(MW)	(pu)	(MW)
G4	1478	0.0771	113.954	0.02756	40.7331
G3	50	0.0704	3.51859	0.03607	1.80298
G2	10	0.0686	0.68531	0.03454	0.34509
G1	250	0.0618	15.45	0.02728	6.81677
G16	101	0.0609	6.1509	0.01238	1.25008
G11	544	0.0401	21.8144	0.0147	7.99611
G9	378	0.0348	13.1544	0.00485	1.83217
G8	378	0.0347	13.1166	0.00927	3.50519
G10	759	0.0347	26.3373	0.01358	10.3095
G6	511	0.0314	16.0454	0.00294	1.50336
G13	543	0.0195	10.5885	0.01231	6.68216
G12	523	0.0192	10.0416	0.01224	6.40204
G15	90	0.0116	1.044	0.00859	0.77337
G5	309	0.0053	1.6377	0.00015	0.04604
G7	50	0.0048	0.24	0	0
G14	445	0	0	0.01686	7.50075
Contribution of Gs to Losses		253.779		97.5	
Contribution of Ds to Losses		-75.0738		0	
Net Contribution		178.705		97.5	

A. Variable (Losses) Charges

Table 2, above, illustrates how the marginal and tracing methods charge for transmission losses as the only variable costs for this network. It was assumed that the tracing-based charges are levied on the generators only. This table shows that marginal pricing of transmission losses leads to over-recovery of variable transmission costs while the tracing charges recovers the exact costs of losses. However, both schemes do not eliminate the necessity to rely on some form of multi-part tariff, which usually involves usage (variable) and access (fixed) charges.

Marginal charges are economically efficient as they reflect the exact impact of both generators and demand centres on the system costs. In the case illustrated in Table 2, generators increase losses while demand centres collectively reduce these losses. The tracing, on the other hand, charges either generators or demand centres (or some split between the two). Changing the marginal generator changes the marginal losses but the ranking of the generators remain the same. The tracing charges are independent of the choice of the marginal node and, hence, are very stable.

However, this way of charging for losses has a direct implication on the social net welfare of society as expressed by the total surplus, which is equal to the sum of producers' surplus (i.e. profit) and consumers' surplus (i.e. the difference between what they are willing to pay and what they are actually paying) [5]. The marginal charges maximize the social net welfare as it produces efficient prices that reflect each user's impact on losses. Thus, these charges can be positive, zero or negative depending on such actions. Changing the marginal node result in only transferring part of the surplus from one group of users to another which keep the total surplus unchanged. This means that regardless of which generator is assumed as the marginal generator, *ex ante* marginal charges preserve the efficient dispatching. However, the serious effects of such allocations on both generators' revenues and loads' payments can be disputed.

On the other hand, the tracing method charges only one group of users but not the other. As a result, positive charges for generators, as in Table 2, means that their revenues are lower. This reduction in generators' revenues result in lower producers' surplus, but the loads' payments and, hence, consumers' surplus remain unchanged. Consequently, the sum of these two effects results in lower total surplus, from the reduction in producers' surplus, which is almost equal to the total losses.

B. Fixed (Access) Charges

Since losses are non-linear, marginal charges recover 178.7MW, which is almost double the total losses. Although it has been suggested that the value of this surplus to be used for remunerating part of the fixed cost, it is insufficient as it normally covers only a fraction of the

required revenues [6]. This over-recovery of losses may not always be acceptable to some users and it may also lead to inefficient operation of the system if the transmission company revenues are not regulated. The tracing charges, on the other hand, recover the exact total losses but this obviously results in less efficient outcomes than those of pricing at the margin.

Nevertheless, even efficient short run charges would not necessarily is a good basis for long-run decisions about the investment in the network and capacity expansion. The need to recover the fixed costs of the transmission network usually requires a two-part tariff scheme involving access (fixed) and variable charges. The drawback of this two-part tariff is that the efficient economic signal of marginal prices is weakened due to the presence of significant sunk costs. These costs are normally recovered through postage stamp charges on the basis of per unit of generation capacity (or maximum demand). These charges ignore locational differences because it considers as if all generators and demand centers are located at the same point on the network. Therefore, the large size of transmission fixed costs would cause access charges to distort the spatial signal of the efficient short-run prices.

Despite the negative implications of these charges on marginal decisions, such two-part tariffs scheme is common practice by many transmission systems. This form of charging may be acceptable in the regulation of public utilities, but in the case of a business with a locational dimension, such as the provision of transmission services, fairness considerations may require that access charges be spatially differentiated. The average charge would clearly obscure the locational signal provided by the marginal cost price. On the other hand, spatially differentiated access fees may strengthen this signal if the method used for designing the access fees provides cost allocation which give similar signals to those of marginal charges.

The tracing method portions the power flows and consequently the losses over every line in the network. This makes it possible to allocate the total transmission costs based on 'the extent of use' through variable (losses) and fixed (access) charges. It has been suggested that the best application of the tracing method is in the allocation of fixed costs as a supplement to the marginal cost price [7]. This has the advantage of incorporating locational signals into cost recovery through access charges. Hence, the capability of the tracing method to set spatially differentiated access charges could reduce this distortion, especially when zonal tracing charges as discussed, above, give a similar spatial signal to that of marginal charges.

8 CONCLUSION

The privatisation of the electricity industry and the separation of generation from transmission and distribu-

tion have made transmission pricing vital to this process. The recent increasing interest by theorists and practitioners alike in the issue of transmission pricing developed from the necessity to find an efficient and practicable way of allocating transmission costs. In contrast to the case of a vertically integrated industry, these two objectives are difficult to reconcile in an unbundled structure, as transmission pricing methods are required to satisfy objectives which are mostly contradictory. There are different methods of transmission pricing ranging from the theoretical (marginal pricing) to the practical (postage stamp).

This paper has explored one of the major features of the marginal method and has conducted a comparison between this method and the new method of electricity tracing, which might be a useful method of allocating transmission costs. The deviation of the tracing method from the ideal method is not large enough, especially for zonal charges, to disregard it. The tracing charges have the advantage of being stable and fair with a reasonable consideration of spatial factors. These attributes are very much needed in dealing with the vexing issue of transmission pricing especially in deregulated electricity markets.

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