

# COMPARISON OF DIFFERENT CONTROL STRUCTURES FOR LYAPUNOV-BASED POWER SYSTEM STABILIZER

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**Abstract:** Recently it has been shown that it is possible to design Power System Stabilizer (PSS) using a non-linear multi-machine system model in conjunction with Lyapunov's direct method. The resulting controller has been shown to be robust. It was also shown that each individual controller acted independently of all the others suggesting that no coordination of settings is necessary. These features should allow a decentralized approach to the design of the PSS. In this paper two control structures implementing the proposed control law have been compared: a hierarchical structure in which the Automatic Voltage Regulator (AVR) is the master controller and the PSS is the slave controller, and a more traditional structure in which the PSS constitutes a supplementary loop to the main AVR. This paper shows that the AVR has to be designed in a different way for each of the structures. Theoretical analysis is supported by the results of simulations tests which highlighted the differences between the way in which the stabilizing signal was obtained for each controller. The tests performed on a multi-machine system model have shown a very good performance of both proposed structures when compared with a classical speed-based PSS.

**Keywords:** power system stability, power system control, synchronous generator excitation

## I. INTRODUCTION

Large interconnected power systems often suffer weakly damped oscillations between synchronous generators. This problem has grown in importance in recent years due to two factors: (i) the load on transmission networks keeps increasing with very limited possibilities of expanding the existing transmission networks; (ii) introduction of liberalised electricity markets has resulted in changes in power transmission patterns. The most cost-effective way of power oscillation damping is by a Power System Stabiliser (PSS), which is a supplementary loop to the main Automatic Voltage Regulator (AVR) controlling the generator excitation voltage. Considerable research effort has been devoted world-wide to the design of PSS. This paper concentrates on an approach in which the PSS was designed using a non-linear system model in conjunction with Lyapunov's direct method [1-3]. The resulting controller has been shown to be robust, as the damping it introduced into the system was insensitive to changes in both the system topology/parameters and the pattern of network flows. It was also shown that each individual controller acted independently of all the others suggesting that no coordination of settings is necessary. These

features should allow a decentralized approach to the design of the PSS. Such a design approach is important for the new competitive market structures where generation is usually separated from transmission and individual generators compete against each other. This makes it difficult to exchange information about parameters of individual generating units. Such information is necessary for a traditional tuning of the stabilizers. With the proposed approach no tuning seems to be necessary. This should also result in savings on commissioning costs.

In this paper two control structures implementing the proposed control law are compared: a hierarchical structure in which the Automatic Voltage Regulator (AVR) is the master controller and the PSS is the slave controller, and a more traditional structure in which the PSS constitutes a supplementary loop to the main AVR. First theoretical analysis is performed which shows that the AVR has to be designed in a different way for each structure. Then results of simulation tests performed on a multi-machine system are shown which compare both proposed controllers with a classical speed-deviation-based PSS.

## II. THEORETICAL BACKGROUND

Point  $\hat{x}$  is the equilibrium point of the dynamic system described by a set of non-linear equations  $\dot{x} = F(x)$  if  $F(\hat{x}) = 0$ . Lyapunov's stability theorem states that this equilibrium point is stable if there is a Lyapunov function  $V(x)$  such that: (i)  $V(x)$  is positive definite with a minimum value at  $\hat{x}$ , and (ii) the time derivative  $\dot{V} = dV/dt$  along the system trajectory  $x(t)$  is negative semi-definite, i.e.  $\dot{V} \leq 0$ . If  $\dot{V} < 0$  then the equilibrium point is asymptotically stable. If  $\dot{V}$  is negative then the function  $V(x)$  decreases with time and tends towards its minimum value, the system equilibrium point  $\hat{x}$ . The more negative the value of  $\dot{V}$  the faster the system returns to the equilibrium point  $\hat{x}$ , i.e. the faster damping of any swings.

The automatic control of any system element, such as synchronous generators, turbines or FACTS devices, influences the value of  $\dot{V}$ . Consequently any given control is optimal, in the sense of a chosen Lyapunov function, if it maximizes the negative value of  $\dot{V}$  at each instant of the transient state. As Lyapunov functions are non-unique, another Lyapunov function may result in another optimal control.

### III. CONTROL LAW

The approach outlined above has been applied to design a PSS using initially a fourth-order non-linear single-machine-infinite-busbar system model [1] and then third-order non-linear multi-machine system model [2]. In the latter case the Lyapunov function was expressed as the sum of the system kinetic energy, potential energy, and a term proportional to the sum of squared deviation of the transient emf for all the machines. It was further proved in [2] that the derivative of the Lyapunov function can be expressed as

$$\dot{V} = -\sum_{i=1}^n D_i \Delta\omega_i^2 - \sum_{i=1}^n \frac{1}{T_{d0i}} \frac{1}{X_{di} - X'_{di}} (E_{qi} - \hat{E}_{qi})^2 + \dot{V}_{E_f} \quad (1)$$

where

$$\dot{V}_{E_f} = \sum_{i=1}^n \frac{1}{T_{d0i}} \frac{1}{X_{di} - X'_{di}} (E_{fi} - \hat{E}_{fi}) (E_{qi} - \hat{E}_{qi}) \quad (2)$$

In these equations the subscript  $i$  relates to the generator number,  $\Delta\omega_i$  is the speed deviation,  $X_{di}$  and  $X'_{di}$  are the synchronous and transient  $d$ -axis generator reactance, respectively,  $E_{qi}$  is the  $q$ -axis synchronous emf,  $E'_{qi}$  is the  $q$ -axis transient emf,  $T_{d0i}$  the open-circuit transient time constant,  $E_{fi}$  the excitation voltage. A “hat” on the top of a symbol corresponds to the post-fault equilibrium point.

The first and the second components in (1) are negative semi-definite and always contribute to the overall system damping. The third component is given by (2) and it is this component that is influenced by the excitation control. Thus the optimal control law must be such that at any instant in the transient state each component of (2) must be negative and maximal. This condition is satisfied by the control law

$$(E_{fi} - \hat{E}_{fi}) = -K_i (E_{qi} - \hat{E}_{qi}) \quad (3)$$

where  $K_i$  is the controller gain. Obviously this law is optimal only for the Lyapunov function used.

### IV. IMPORTANT FEATURES

The first important feature of the control law (3) is that the function  $\dot{V}$ , equation (1), is independent of the network parameters. Recall that  $\dot{V}$  effectively determines the system damping. This would suggest that the contribution of each generator, controlled using (3), to the overall system damping is insensitive to changes in the network topology/parameters or the pattern of network flows. This is very important as it suggests that the proposed stabilizer is robust in that it does not need re-tuning following network changes.

The second important feature of control law (3) is that each generator contributes an independent component into  $\dot{V}$  given by (1), with no cross-coupling terms between generators. In other words each individual controller contributes a damping term independent of all the other controllers. This very important feature of the proposed controller is referred to as *the*

*additivity of damping*. In traditional AVR+PSS systems the settings of individual controllers must be co-ordinated as any controller may influence any other controller in the system. In contrast, the additivity of damping for the proposed controllers suggests that their settings need not be co-ordinated.

The robustness of the proposed stabilizer in conjunction with the additivity of damping are very important features of the proposed control as they allow a decentralized approach to the design of stabilizing controllers. The parameters of the proposed AVR+PSS system can be determined in a decentralized way, without considering the impact of other controllers or system conditions. This feature of the proposed controller, which is unique among competing AVR+PSS design methodologies, should reduce the commissioning costs as no system-wide studies are necessary. It is also compatible with the new market structure in which individual generators compete against each other and may be unwilling to disclose detailed information about their generators and control systems. Such information is necessary for optimal tuning of traditional AVR+PSS systems.

### V. ALTERNATIVE CONTROL STRUCTURES

This section assumes the use of a static exciter which can be represented as a proportional block. Synchronous emf  $E_f$  is then proportional to the excitation voltage  $V_E$  produced by the AVR and, in per-unit,  $V_E = E_f$ . This argument is valid only for the static exciter. For other types of exciter an additional transfer function would have to be included in the block diagram to compensate for the equivalent transfer function of the exciter.

#### A. Hierarchical structure

The control law defined by (3) can be re-written as

$$E_{fi}(t) = \hat{E}_{fi} - K_i (E_{qi}(t) - \hat{E}_{qi}) \quad (4)$$

where

$$\hat{E}_{fi} = \hat{E}_{qi} = E_{q\ ref i} \quad (5)$$

defines the post-fault steady-state synchronous emf. As  $E_{qi}(t) = X_{adi} i_{fi}(t)$ , where  $X_{ad}$  is the  $d$ -axis armature reaction reactance and  $i_f$  is the field current, the control law (3) can be expressed as:

$$E_{fi}(t) = E_{q\ ref i} - K_i [X_{adi} i_{fi}(t) - E_{q\ ref i}] \quad (6)$$

Fig. 1 shows the functional block diagram of a controller executing control strategy (6). Symbol ST denotes a static exciter. The lower part of this diagram, denoted as PSS, generates signal  $K \Delta E_q$  that is proportional to the change in the generator synchronous emf with respect to the post-fault equilibrium value  $E_{q\ ref}$ . This signal is subtracted from the reference value  $E_{q\ ref}$  determined by the AVR shown in the upper part of the diagram. The AVR sets the reference value  $E_{q\ ref}$  depending on the required terminal voltage  $V_{ref}$  and the system loading condition.

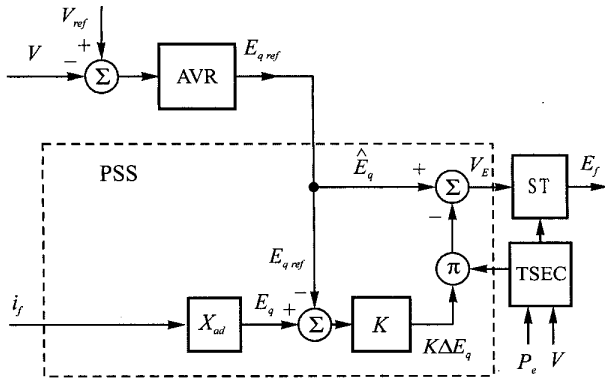


Fig. 1 Block diagram of the hierarchical structure

The structure of the proposed controller is hierarchical and of the master-slave type. The slave (primary) controller is the PSS and the master (secondary) controller is the AVR. The PSS has two input signals: (i) the reference value of the synchronous emf provided by the AVR and (ii) a feedback signal provided by the field current. The PSS does not contain any phase compensation lead-lag elements. It is basically a proportional controller as the change in the excitation voltage is proportional to the change in the synchronous emf (or the field current).

It is important to appreciate that the control law (3) is valid only in the post-disturbance state. During a short-circuit the generator rotor accelerates and correct control action should increase the excitation voltage  $E_f$  in order to increase the available deceleration area. However a short-circuit causes the generator synchronous emf  $E_q$  to increase suddenly [4, 5] and the proposed controllers would counteract this by reducing the excitation voltage instead of increasing it. In order to deal with such situations the supplementary controller shown in Fig. 1 is equipped with an additional logic circuit referred to as *transient stability excitation control* (TSEC). The aim of the TSEC is to detect a short-circuit and force the maximum excitation voltage. Often a similar logic circuit is used in a traditional PSS [4].

There are many ways of designing the TSEC logic. In a simple version the TSEC detects the simultaneous presence of a large voltage error  $\Delta V$  and a large acceleration power  $\Delta P$ . In the case of a short circuit in the network both the real power and generator terminal voltage decrease rapidly and the increments  $\Delta P$  and  $\Delta V$  are both large and negative. In the case of a power swing (forward or backward) the increments  $\Delta P$  and  $\Delta V$  have opposite sign. This enables the TSEC to distinguish between a short-circuit and a power swing.

### B. Structure with a supplementary loop

As  $E_{q\ ref i} = X_{adi} i_{f\ ref i}$ , equation (6) can be re-written as:

$$E_{fi}(t) = E_{q\ ref i} - K_i X_{adi} \Delta i_{fi}(t) \quad (7)$$

where  $\Delta i_{fi}(t) = i_{fi}(t) - i_{f\ ref i}$  is the increment in the field current obtained using a “wash-out” element which eliminates

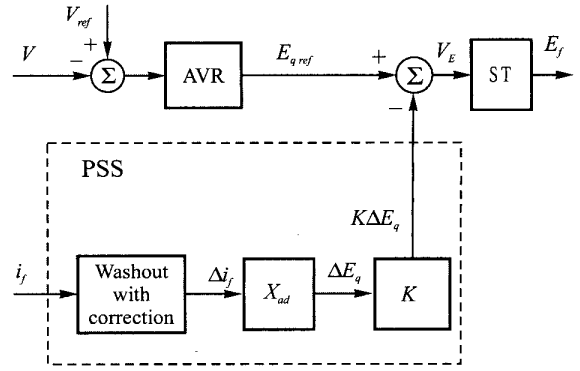


Fig. 2 Block diagram of the structure with supplementary loop.

the DC component from the field current. Fig. 2 shows the functional block diagram of the AVR+PSS system executing strategy (7). This system has a traditional structure as the AVR is the main controller and the PSS is a supplementary controller.

Note that the supplementary PSS contains no TSEC block. The reason for this will be explained in Section VII.

### C. Additional filters

The controllers shown in Fig. 1 and Fig. 2 use the field current as their input signal as the field current is proportional to  $E_q$ . It should be noted, however, that a short-circuit produces dc offset in the armature current which induces a rapidly decaying 50 (or 60) Hz component in the field current. Moreover, a static exciter using commutation of an ac source will also produce persistent ripples in the field current. All these fast-changing signals must be filtered out using a low-pass filter which has not been shown in Fig. 1 and Fig. 2.

## VI. CHOOSING THE PARAMETERS OF THE CONTROLLERS

According to the control law (3) the higher the gain  $K$  the better the damping. However, because of the interactions between the AVR and PSS, the gain of the PSS and of the AVR has to be chosen differently for the hierarchical and the supplementary structure.

In both of the control structures the output signal from the AVR,  $E_{q\ ref}$ , plays two roles. On one hand it maintains the desired terminal voltage  $V_{ref}$  while on the other hand it provides the reference value for the PSS. Thus  $E_{q\ ref}$  should not change as a consequence of voltage fluctuations following a power swing. To achieve this, the AVR should have a low effective gain at the swing frequency of typically 0.2-2 Hz. This can be achieved if the AVR is a PI regulator, or a proportional regulator with an appropriate transient gain reduction (TGR).

The supplementary structure shown in Fig. 2 is very much like the traditional AVR+PSS system. In order to get a good voltage regulation, the gain of the AVR has to be high while a TGR block reduces the effective gain during power swings. In this structure the PSS signal is subtracted from the AVR signal

to give the exciter voltage  $V_E$ . Thus the gain of the PSS cannot be too high as this would reduce the effective gain of the AVR+PSS system for the voltage regulation purposes. Trying to satisfy the conflicting needs of the voltage regulation and effective damping resulted in the choice of the following transfer function of the AVR in the supplementary structure:

$$G_{AVR\text{ suppl}}(s) = 100 \frac{1+s}{1+10s} \quad (8)$$

The corresponding gain of the PSS was chosen to be 6.66 while the transfer function of the washout element was chosen to be:

$$G_W(s) = \frac{10s}{1+10s} \frac{1+0.05s}{1+0.02s} \frac{1+3s}{1+5.4s} \quad (9)$$

The situation is different in the hierarchical structure shown in Fig. 1. In this structure the AVR and the PSS act not in parallel but in series. Consequently the overall gain for the voltage control purposes can be shared between the two controllers. This allows to chose a low gain for the AVR (in order to keep  $E_{q\text{ ref}}$  relatively constant during power swings) and a high gain for the PSS (in order to get a good damping of the power swings). In our simulations we have assumed the AVR to be a PI controller with the gain of 2 and the integration time constant of 1s. The corresponding gain of the PSS was chosen to be 10. These values resulted in a good voltage regulation and a good damping.

There is also one important difference between the way the control signal is created in both structures. It is important to appreciate that  $\hat{E}_{fi} = \hat{E}_{qi}$  in (3). Consequently the reference value, with respect to which the deviation  $\Delta E_q$  is calculated, must be equal to  $E_{q\text{ ref}}$ . This is the case in the hierarchical structure shown in Fig. 1 but not necessarily in the supplementary structure shown in Fig. 2. In the latter case the signal  $\Delta E_q$  is obtained by filtering out the DC signal from the field current  $i_f$  rather than subtracting  $i_{f\text{ ref}}$  from  $i_f$ . Thus the supplementary structure obeys the control law (3) only if  $i_{f\text{ ref}}$  is equal to the DC component in  $i_f$  which may not always be the case. Moreover, the washout block shown in Fig. 2 introduces phase shifts which further distort the signal.

The overall conclusion is then that the hierarchical structure shown in Fig. 1 seems to be superior from a theoretical viewpoint as: (i) it allows using a higher gain for the PSS; and (ii) the signal  $\Delta E_q$  is created in a direct way without introducing any phase shifts due to a washout block.

## VII. SIMULATION RESULTS

Simulation results are presented for the four-machine system shown in Fig. 4 [4]. The system consists of two areas and each area has two generators. The frequency of the two local modes is about 1 Hz while the frequency of the inter-area mode is about 0.5 Hz.

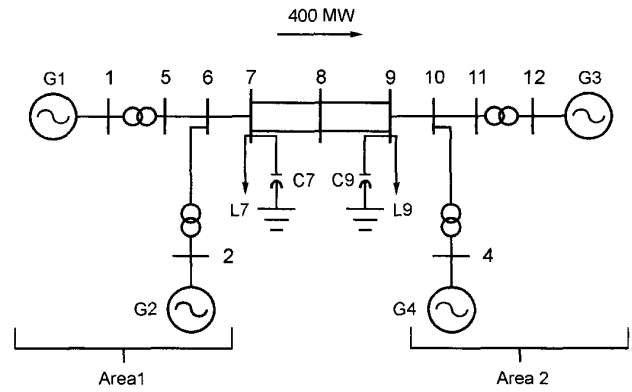
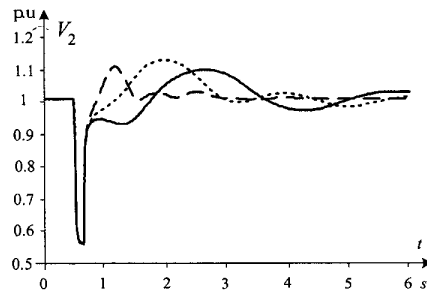
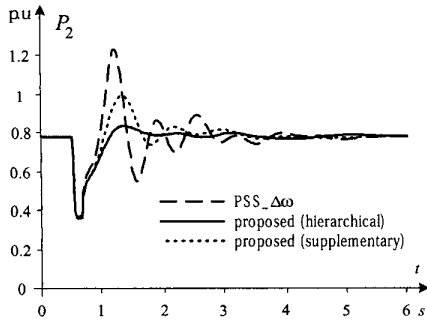


Fig. 3 Diagram of the four-machine test system [1].

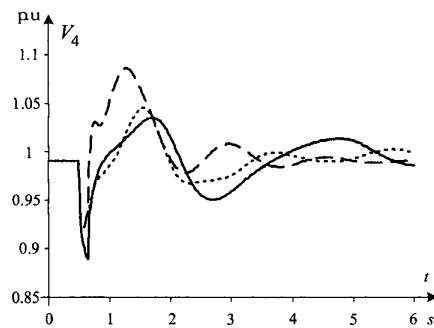
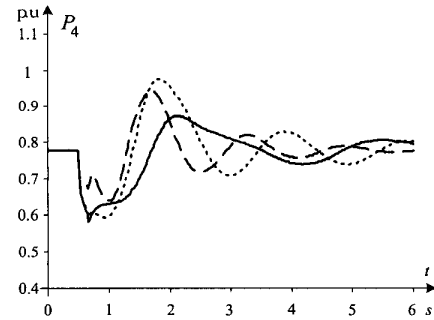
Three excitation control schemes have been compared by simulation. The first scheme denoted as "PSS\_Δω" corresponds to a classical PSS based on speed deviation Δω. The structure and the parameters of the controllers have been taken from [4]. The second scheme, denoted as "proposed (hierarchical)", corresponds to the hierarchical excitation controller of Fig. 1. The third excitation control scheme, denoted as "proposed (supplementary)", is the proposed excitation controller with the supplementary loop, Fig. 2. The PSS controllers in all three cases have the input deadzone of ±0.005 and the output limiter of ±1.75. The AVR has the input deadzone of ±0.05 while the output limit of the excitation voltage is  $E_{f\text{ max}} = 6.5$ ,  $E_{f\text{ min}} = -6$ .

The sixth-order generator model has been used in all the simulations so as to include sub-transient effects in both axes. The generator and network resistances have also been included. The loads have been modeled as constant real and reactive power demands. A number of different types of tests have been conducted but here only the results of a temporary short circuit at node 5 (fault close to generators 1 and 2) will be shown. The fault duration was assumed to be 150 ms.

Damping of local swings will be shown using the example of generator G2 (close to the fault) and generator G4 (away from the fault). Fig. 4 shows the power and voltage swings of generator G2. Clearly the hierarchical controller achieved the best damping of power swings (top diagram in Fig. 4). Both the first overshoot and the settling time were greatly reduced when compared to the classical Δω-based PSS. The proposed supplementary controller performed better than the Δω-based PSS but worse than the proposed hierarchical PSS. When the voltage swings are analyzed (lower diagram in Fig. 4), Δω-based PSS performed slightly better than both proposed systems. The first overshoot was about the same for all three systems but the settling time was shorter.



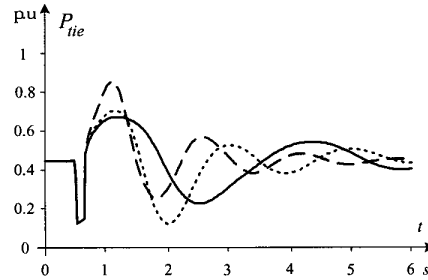
**Fig. 4 Damping of local swings: generator G2.**



**Fig. 5 Damping of local swings: generator G4.**

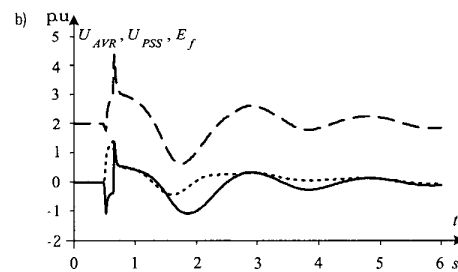
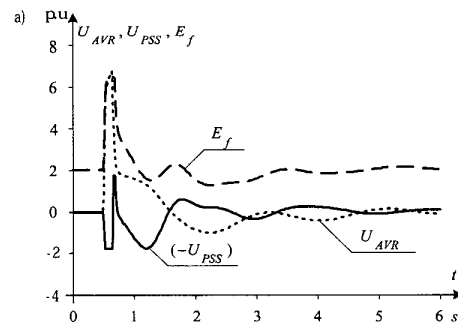
Fig. 5 shows the power and voltage swings for generator G4 which was further away from the fault. Again damping of the power swings was the best for the proposed hierarchical system, while the proposed supplementary and the  $\Delta\omega$ -based PSS performed similarly. Interestingly, the voltage overshoot was the worst for the  $\Delta\omega$ -based PSS. Quite noticeably, both

proposed systems reduced the frequency of power swings when compared to the  $\Delta\omega$ -based PSS. The proposed hierarchical PSS was also much slower than the supplementary PSS.



**Fig. 6 Damping of inter-area swings.**

Fig. 6 shows damping of inter-area power swings between nodes 7 and 9. The  $\Delta\omega$ -based PSS suffered the worst first overshoot but it settled down slightly faster than both proposed stabilizers. Both the supplementary and the hierarchical proposed stabilizers produced a similar first overshoot but the hierarchical system provided a better damping. Again, the proposed hierarchical PSS significantly reduced the frequency of power swings.



**Fig. 7 Supplementary controller signals: (a) generator G2; (b) generator G4.**

In order to explain why TSEC block is not necessary when the supplementary proposed system is used, Fig. 7 shows the control signals in generators G2 and G4. Signal  $U_{AVR}$  is the

change in the output of AVR – see Fig. 2. Signal  $(-U_{PSS})$  is the output of the PSS, equal to  $K\Delta E_q$ . Signal  $E_f$  is the output excitation voltage:

$$E_f = V_E = E_{q\text{ref}0} + U_{AVR} - U_{PSS} \quad (10)$$

where  $E_{q\text{ref}0}$  is the initial value of  $E_{q\text{ref}}$ .

Let us first consider Fig. 7a which shows the control signals of generator G2 (close to the fault). During the fault the signal  $U_{AVR}$ , shown by a dotted line, increases which is a correct stabilizing action. Unfortunately, as mentioned in Section V.A, the PSS counteracts this by producing a negative signal  $(-U_{PSS})$  shown by the solid line. However the output limiter of the PSS reduces the drop in the value of PSS signal to  $-1.75$  pu so that the resultant  $E_f$  increases anyway, almost to its upper limit (6.5 pu). Consequently the proposed PSS does not deteriorate the stabilizing action of the control system during the fault and the TSEC block is not necessary. To justify this note that even if the TSEC block was used, making the signal  $(-U_{PSS})$  positive during the fault, the AVR output limiter would limit  $E_f$  to almost the same value of 6.5.

Now consider Fig. 7b which shows the control signals of generator G4 (remote from the fault). Now the changes in the control signals are not high and the limiters are not activated. Also the  $(-U_{PSS})$  value does drop during the fault, the magnitude of changes is small and it does not influence the system stability. Hence the TSEC block is again not necessary.

To confirm the hypothesis that the TSEC system is not necessary for the proposed supplementary system, simulations have been performed when the TSEC block was included. The resulting damping of power swings was not better when compared to the case without TSEC. Obviously the TSEC block is necessary when the hierarchical system is used.

The general conclusion therefore is that although the damping enforced by the proposed hierarchical system seems to be superior to that of the proposed supplementary system, the latter is simpler as it does not need the TSEC block. This is a significant advantage as designing a properly functioning TSEC block is not easy.

### VIII. CONCLUSIONS

It has been shown recently that it is possible to design Power System Stabilizer (PSS) using a non-linear multi-machine system model in conjunction with Lyapunov's direct method. The resulting controller has been shown to be robust. It was also shown that each individual controller acted independently of all the others suggesting that no coordination of settings is necessary. These features should allow a decentralized approach to the design of the PSS.

In this paper two control structures implementing the proposed control law have been compared: a hierarchical structure in which the Automatic Voltage Regulator (AVR) is the master controller and the PSS is the slave controller, and a more traditional structure in which the PSS constitutes a supplementary loop to the main AVR. It has been shown that the AVR has to be designed in a different way for each of the structures. Theoretical analysis, supported by the results of

simulations tests, highlighted the differences between the way in which the stabilizing signal was obtained for each controller. It has been shown that the damping enforced by the proposed hierarchical system seems to be superior to that due to the proposed supplementary system. However the latter is simpler as it does not need the Transient Stability Excitation Control (TSEC) enforcing high excitation level during a fault. This is a significant advantage as designing a properly functioning TSEC block is not easy.

The tests performed on a multi-machine system model have shown a very good performance of both proposed structures when compared with a classical speed-based PSS.

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