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A Methodology for Allocating Transmission Losses Due to Cross-Border Trades

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***Abstract*—Transmission pricing of cross-border trades is often difficult since individual countries may use incompatible internal transmission pricing regimes, and they are usually unwilling to disclose any sensitive information about their own systems. In this paper, the use of a tracing methodology is proposed to overcome these problems using loss allocation problem as an example. Firstly, a modification of the tracing methodology is presented in which the loss is allocated in a direct way. Then a unifying tracing-based methodology of transmission pricing for cross-border trades is proposed. The only data required are the flows in the tie-lines and the charges to be applied to the border nodes in each country. No information is required about individual transactions, load/generation profiles or internal networks. The methodology is illustrated on the IEEE 118 node network divided into four areas, each with a different internal transmission pricing methodology. The proposed methodology is shown to be simple, transparent and very fast and it can deal effectively with cyclic flows.**

***Index Terms*—Interconnected power systems, power system economics.**

I. INTRODUCTION

OVER the past decade, many countries in the world have started restructuring their electricity supply industries to make them more lean and efficient. Restructuring requires a number of difficult problems to be resolved but one of the most controversial ones is the development of an efficient transmission pricing methodology.

A proper transmission pricing regime should satisfy a number of requirements which are often contradictory [1], [2]. Two apparently conflicting requirements are that

- competition should be promoted by presenting the network user a predictable, stable and practical-to-apply framework of charges;
- transmission prices should provide signals toward the efficient use, operation and expansion of the network.

Different countries place varying emphasis on each of these requirements and, consequently, there are virtually no two countries in the world that operate identical transmission pricing regimes. This creates a problem whenever a trade is attempted which crosses a country border (or control area border within a country). Examples of this are the USA and the UCTE network in Europe. In both cases the interconnected networks consist of a number of control areas (countries in Europe) with often incompatible internal transmission pricing regimes. This incompatibility is a serious impediment to the ability to transact interchanges of power between control areas (or across borders in Europe). This is especially difficult when the trading parties are not in neighboring countries or control areas. One way of dealing with that problem is to try to make internal market arrangements in individual control areas (or countries) to be as close to each other as possible. This approach is being tried in the USA with the Standard Market Design. However any moves toward standardization inevitably meet strong opposition from utilities (or countries) protecting their independence. Especially in Europe, standardization of market design would be probably not possible to achieve. To address that problem, this paper advocates the use of a unifying framework for cross-border transmission charging rather than forcing the countries to adopt the same internal pricing methodologies. The proposed methodology would be applied to cross-border trades only and would accommodate different transmission pricing regimes within each country. The paper concentrates on transmission loss allocation and pricing but the assertions and findings are equally relevant to other transmission costs.

Marginal pricing of losses provides a theoretically optimal signal for operation and investment [3]. However, nodal marginal loss factors depend on the choice of the slack bus and that may prove to be contentious in a large multi-national network [4]. Often the problem is considered as the "fair" allocation of losses, rather than their pricing. The definition of "fairness" is nonunique and gives rise to a number of different possible approaches. Here only some of them are briefly discussed. In [5], the transmission loss due to a transaction is calculated with high accuracy using the Hessian matrix. In [6], the loss is allocated to individual transactions using the dc network model. In [7], the loss is allocated by integrating each transaction along an assumed path. In [8], allocation factors are derived using the first derivative of loss function. A common feature of all of these methodologies is that they require a detailed knowledge of the mathematical model of the network and of individual transactions. This information is unlikely to be available in an interconnected system consisting of many areas (or countries) because of the sheer system size and the unwillingness of individual utilities to disclose commercially sensitive information about their networks, customers and transactions.

The approach presented in this paper addresses the above problem and it follows the general rules agreed in European Union (EU) for establishment of cross-border tariffs [2].

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The transmission tariffs for cross-border trades in EU will be entry/exit, rather than transaction, based. The payment of compensation for costs of through transfers of energy will be made to the Transmission System Operators (TSO) rather than to individual network users. This paper shows how the tracing methodology [9], also referred to as the "Average Participations Method" [2], [4], could be used for such inter-TSO compensation payments. Each country may be using different internal transmission loss charging methodologies and no information is required about individual transactions, load/generation profiles or internal networks.

The paper is organized as follows. Section II introduces a modification of the tracing methodology which allows the losses to be allocated directly. Section III illustrates the proposed methodology by means of a simple example. Section IV shows how the methodology can be used to allocate compensation payments rather than the losses themselves. Section V shows how loss allocation may be resolved in the presence of cyclic flows while Section VI illustrates and validates the methodology using the IEEE 118 node network. Section VII concludes that this is a robust technique for allocating losses and charges.

II. DIRECT LOSS ALLOCATION USING TRACING METHODOLOGY

A tracing methodology can be seen as a compromise between an economically optimal marginal pricing process and simple averaged (postage stamp) pricing techniques [10]. It can be conducted either by following the noncyclic directed graph (digraph) of flows [4], [12] or by solving linear equations [9]. Although the former is intuitively easier to understand, in this paper the latter will be presented as it is mathematically more compact and it deals more easily with the cyclic flows discussed in Section V.

At the outset we will introduce a modification to the standard tracing methodology [9] such that the loss allocation is calculated directly and in a simpler way.

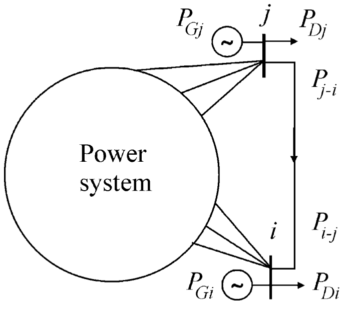

Fig. 1. Illustration to tracing-based loss allocation.

Consider Fig. 1. Line j-i connects a sending node j with a receiving node i. Both nodes may be connected to the rest of the network by a number of lines. Conceptually, we will consider the flow of power from generators to loads according to the directed graph (digraph) of flows, calculate transmission losses accumulated along the way, and allocate them to flows. Let us start from the loss allocation to the loads, when we will look at losses accumulated *upstream* from a given node in the digraph of flows.

Consider allocating losses to the outflows from node j, i.e., line flow P_{j-i} and demand P_{Dj}. The allocation must include all the losses that have been accumulated in all the lines *upstream* from node j in the digraph of flows. Assuming that the loss is allocated proportionally to the value of a flow from the node, it can be calculated as

$$\Delta P_{j-i}^{(u)} = \frac{|P_{j-i}|}{P_j}\Delta P_j^{(u)} \quad \text{and } \Delta P_{Dj} = \frac{P_{Dj}}{P_j}\Delta P_j^{(u)} \tag{1}$$

where $\Delta P_{j-i}^{(u)}$ is the loss allocated to line flow $P_{j-i}, \Delta P_{Dj}$ is the loss allocated to demand P_{Dj}, P_j is the total flow through node j (i.e., the sum of inflows or outflows) and $\Delta P_j^{(u)}$ is the unknown upstream loss allocated to the sending node j.

The upstream loss $\Delta P_i^{(u)}$ allocated to the receiving node i may now be calculated. It will be equal to the sum of the *actual* transmission loss in line $j-i, \Delta P_{j-i} = |P_{j-i}| - |P_{i-j}|$, and the unknown upstream loss allocated to the sending-end flow, $\Delta P_{j-i}^{(u)}$ expressed by the first equation of (1). Summing contributions from all the lines supplying node i gives

$$\Delta P_i^{(u)} = \sum_{j\in\alpha_i^{(u)}} \Delta P_{j-i} + \sum_{j\in\alpha_i^{(u)}} \frac{|P_{j-i}|}{P_j}\Delta P_j^{(u)} \tag{2}$$

where $\alpha_i^{(u)}$ is the set of nodes directly supplying node i. Equation (2) can be expressed in a matrix form after moving the second component from the right-hand-side in (2) to the left-hand side and combining terms

$$\mathbf{A}_u \Delta\mathbf{P}^{(u)} = \Delta\mathbf{P}_\Sigma^{(u)} \tag{3}$$

where $\Delta\mathbf{P}^{(u)}$ is the vector of all unknowns $\Delta P_j^{(u)}$ for $j = 1, 2 \ldots n, n$ is the number of nodes, $\Delta\mathbf{P}_\Sigma^{(u)}$ is a vector with its i-th element equal to $\sum_{j\in\alpha_i^{(u)}} \Delta P_{i-j}$ (the sum of the actual transmission losses in all the lines supplying node i) and $\mathbf{A}_u$ is the *upstream distribution matrix* defined as

$$[\mathbf{A}_u]_{ij} = \begin{cases} 1, & \text{for } i = j \\ -|P_{j-i}|/P_j, & \text{for } j \in \alpha_i^{(u)} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Solving (3) gives the unknown vector $\Delta\mathbf{P}^{(u)}$ and the final allocation of the total transmission loss to individual loads is obtained from the second equation of (1). Section VI shows an example illustrating the methodology.

For the loss allocation to generators, we will look at losses accumulated *downstream* from a given node. Let $\Delta P_i^{(d)}$ be the unknown downstream transmission loss allocated to node i which has been accumulated downstream from node i in the digraph of network flows. Using a similar reasoning as before, $\Delta P_j^{(d)}$ at the supplying node j can be calculated as the sum of *actual* transmission losses in all the lines supplied from j plus the *downstream* losses $\Delta P_i^{(d)}$ allocated to receiving-end flows

$$\Delta P_j^{(d)} = \sum_{i\in\alpha_j^{(d)}} \Delta P_{j-i} + \sum_{i\in\alpha_j^{(d)}} \frac{|P_{i-j}|}{P_i}\Delta P_i^{(d)} \tag{5}$$

where $\alpha_j^{(d)}$ is the set of nodes supplied directly from node j.

Equation (5) can be expressed in matrix form after moving the second component from the right-hand-side of (5) to the left-hand side and combining terms

$$\mathbf{A}_d \Delta\mathbf{P}^{(d)} = \Delta\mathbf{P}_\Sigma^{(d)} \tag{6}$$

where $\Delta\mathbf{P}^{(d)}$ is the vector of all $\Delta P_j^{(d)}$ for $j = 1, 2 \ldots n$, $\Delta\mathbf{P}_\Sigma^{(d)}$ is a vector with its jth element equal to the sum of losses in all the lines supplied directly from node j, and $\mathbf{A}_d$ is the *downstream distribution matrix* with elements

$$[\mathbf{A}_d]_{ji} = \begin{cases} 1, & \text{for } i = j \\ -|P_{i-j}|/P_i, & \text{for } i \in \alpha_j^{(d)} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Now ΔP_{Gj}, the loss allocated to generator j, can be calculated in proportion to flows as

$$\Delta P_{Gj} = \frac{P_{Gj}}{P_j} \Delta P_j^{(d)} \tag{8}$$

where P_{Gj} is generation at node j.

Section VI will show an example illustrating the methodology. Proportionality assumption, which has been used in the above derivations, can be justified using the cooperative game theory [11].

III. TRACING-BASED LOSS ALLOCATION DUE TO CROSS-BORDER TRADES

For tracing purposes, each country in the interconnected network will be represented by a single node, referred to as the *supernode*, with a net import or export. The net import/export is equal to the balance of internal generation and demand (including losses), effectively a balance of all the imports and exports. Tracing methodology, derived in the previous section, will be then used to allocate the losses to net importers (treated as the loads) and net exporters, treated as the generators.

The alternative approach [2], [4], would be to perform a tracing allocation using the detailed representation of the whole interconnected network, without combining internal networks into equivalent supernodes. This is obviously possible but, in our opinion, suffers from a number of disadvantages which are listed below.

A. Avoidance of Disclosure Commercially Sensitive Information

Obviously, a full detailed representation of all the connections in each control area would provide a more accurate picture of power flows and, arguably, a more accurate loss allocation [4]. The disadvantage is that it would require detailed models of internal networks and knowledge of individual nodal injections. Such information could be deemed commercially sensitive and experience shows that the countries might be unwilling to disclose it. Moreover, such a methodology would only be compatible with internal loss allocation methodologies in each country if they also used tracing techniques. Realistically it is unlikely that all the countries would use the same internal loss allocation methodology, whether tracing or any other. Representing internal electricity networks as supernodes overcomes all of the above problems.

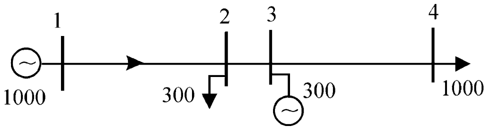

Fig. 2. Example of a balanced group.

B. Subsidiarity

Another major advantage of the supernode approach is that allocation of transmission costs within each country is left to their system operators. This is based on the subsidiarity principle, i.e., that a central (i.e., pan-European) authority should perform only those tasks which cannot be performed at a more local (i.e., country) level, and that principle is considered to be essential in the European Union. For cross-border trades, it is only important to disclose how the internal loss charges are allocated to the border nodes in each country, where an importing node is treated as a generator while an exporting node is treated as a demand. These costs, together with the transmission costs of tie-lines, may be treated as the costs of cross-border trades. Tracing methodology takes those costs and distributes them to those net importing and exporting countries which are deemed to contribute. Hence the internal connections within each country are implicitly taken into account but without explicitly disclosing any internal network connections or load/generation profiles. This is one of the main advantages of the proposed new methodology.

C. No Charges to Countries With Balanced Generation and Demand

Another important advantage of the supernode approach is that it prevents allocation of charges to a transit country with balanced internal generation and demand. Obviously such a country would be entitled to compensations for the losses caused by trades crossing its borders, but it should not be allocated any charges as its net export/import is zero. When a full network model of interconnected network is used, a country with balanced internal generation and demand could be allocated some charges.

D. Better Consideration of the Impact of a Given Country

There is also an additional reason why the supernode model seems to be superior. Consider Fig. 2 which shows a transfer of 1000 MW from node 1 to node 4. Along the way there is a balanced group, consisting of a load at node 2 and a generation at node 3. Nodes 2 and 3 are closely connected, but in turn are connected by long lines to nodes 1 and 4. A similar example was discussed in [4]. Tracing allocates $300/1000$ of the cost of losses in line 1–2 to the load at node 2 and $300/1000$ of the cost of losses in line 3–4 to the generator at node 3. If the load at node 2 and generator at node 3 are considered to be a balanced group (or a country), the net charges to them include a portion of the cost of the losses in the long (and expensive in terms of losses) lines 1–2 and 3–4 but no component of the cost of losses in line 2–3 which happens to connect them. However if the balanced group at nodes 2 and 3 was disconnected, the flows in lines 1–2 and 3–4 would not change, while the flow in line 2–3 would increase by 300 MW. Therefore tracing seems

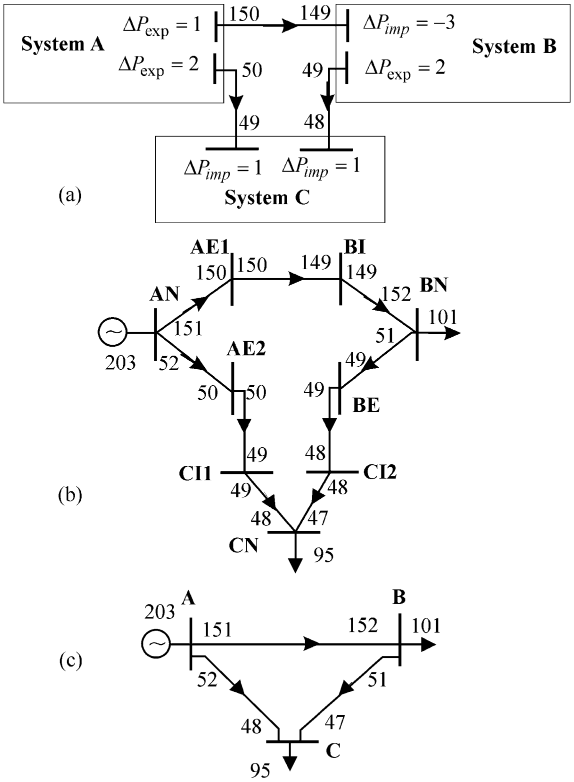

Fig. 3. Example system: (a) flows and losses; (b) equivalent digraph for tracing; and (c) simplified digraph.

to produce unreasonable results when considering the *impact* of a balanced group of closely connected generators and loads on the external network. If such a group was aggregated to consider only its net effect on the network, the balanced group would not be allocated any costs of losses in external lines 1–2 and 3–4. This, in essence, is the supernode approach which relies on netting out exports and imports. Therefore the conclusion is that the supernode approach considers better the impact a given country has on the whole interconnected network.

E. Example

To illustrate the methodology consider Fig. 3(a). Country A has a net export of 200 MW, country B a net import of $149 - 49 = 100$ MW and country C net import of 97 MW. The type of methodology used for the internal loss allocation within each country is irrelevant but, for illustration, marginal loss allocation is assumed. For country B the loss allocated to the import is $\Delta \mathrm{P}_{\mathrm{imp}} = -3$ MW (i.e., the import creates a counterflow and the TSO in B is prepared to pay the equivalent of 3 MW to the users causing it). The loss allocated to the export in B is $\Delta \mathrm{P}_{\mathrm{exp}} = 2$ MW (i.e., the users responsible for this export have to pay equivalent of 2 MW to the TSO in B to compensate for losses). Obviously there may be many import/export nodes in a country, each with its own loss allocation. The country's net injection is equal to the balance of imports and exports plus the balance of the loss charges allocated to the imports and exports, i.e., $49 - 149 - 3 + 2 = -101$, where the minus sign means that it is import.

The digraph corresponding to the cross-border flows is shown in Fig. 3(b). Nodes BI (import in B) and BE (export in B) are the actual border nodes while node BN (net B) is the fictitious node responsible for the net import/export. With many export/import nodes in the country, there would be many export/import branches connected to node BN. The power loss in fictitious branch BI-BN is equal to the loss allocated to the import (note it is negative in this case) while the power loss in the fictitious branch BN-BE is equal to the loss allocated to the export.

TABLE I
LOSS ALLOCATION AND COMPENSATION IN EXAMPLE NETWORK

	System A	System B	System C	Σ
Net export	200	-100	-97	3
Loss compensation	1+2=3	2-3=-1	1+1=2	4
Net injection	203	-101	-95	7
Loss charge	3.5	-0.33	3.83	7
Net charge	0.5	0.67	1.83	3

Country A is a net exporter with two export nodes. Its graph is created similarly to country B. The loss allocated to the exports is shown as a loss in the fictitious lines AN-AE1 and AN-AE2. Country C is a net importer. Its graph is created in a similar manner as that for country B. The only difference is that C imports power using two lines.

The graph shown in Fig. 3(b) contains the actual tie-lines (AE1-BI, AE2-CI1, BE-CI2) and a number of fictitious lines. This graph can be simplified by combining the series-connected branches as shown in Fig. 3(c). The resulting total loss to be allocated is 7 MW which is different from the actual sum of losses (3 MW) as the methodology allocates the sum of actual transmission losses sustained in tie-lines (3 MW) and the loss allocations to import and exports in each country ($1+2-3+2+1+1 = 4$ MW).

The tracing algorithm described in Section II may now be applied to the flows in Fig. 3(c). The net exports are treated as the output of generation sources while the net imports are treated as the loads. The input vectors are then: $\mathbf{P} = [203\ 152\ 95]^T$, $\mathbf{P}_{\Sigma}^{(u)} = [0\ -1\ (4+4)]^T, \mathbf{P}_{\Sigma}^{(d)} = [(4-1)\ 4\ 0]^T$ and the downstream and upstream distribution matrices are

$$\mathbf{A}_u = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-151}{203} & 1 & 0 \\ \frac{-52}{203} & \frac{-51}{152} & 1 \end{bmatrix} \quad \mathbf{A}_d = \begin{bmatrix} 1 & \frac{-152}{152} & \frac{-48}{95} \\ 0 & 1 & \frac{-47}{95} \\ 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Solving (3) and (6) gives the following result: $\Delta\mathbf{P}^{(u)} = [0\ \ -1\ \ 7.6645]^T, \Delta\mathbf{P}^{(d)} = [7\ \ 4\ \ 0]^T$. Let us assume that the losses are allocated in the ratio 50:50 between the net imports and exports. Then, using (8), the loss allocated to the net export in A is $7 \times (203/203)/2 = 3.5$ MW. The other countries are net importers and their loss allocations, using (1), would be $-1 \times (101/152)/2 = -0.3322$ MW and $7.6645 \times (95/95)/2 = 3.8322$ MW, respectively.

Table I illustrates the result showing both the loss charges and compensations due to cross border trades. The sum of net exports/imports (first row) gives the sum of losses in tie-lines. Loss compensations (second row) are credits due to cross-border trades calculated as the sum of internal loss allocations to imports/exports shown in Fig. 3(a). Net injections [third row, also shown in Fig. 3(c)] are equal to the sum of the first and second row. Loss charges (penultimate row) are loss allocations to net importers/exporters calculated using the proposed tracing-based methodology. Net charges (last row in Table I) are equal to the difference between the charges (penultimate row) and credits due to compensations (second row). As proposed by the Council of European Energy Regulators [2], the net country charge should be debited to all the loads in importing countries and all the generators in exporting countries while the net revenues should be credited to all the loads in exporting countries and all the generators in importing countries. The sum of net charges in all countries is equal to net losses in tie-lines.

IV. TRACING LOSS PAYMENTS RATHER THEN MEGAWATTS

It is fundamentally important to establish the price of 1 MW of losses, as the price of energy may be different in each country. The proposed loss allocation scheme allocates losses in terms of MW while to determine compensation payments a reference price is needed. One simple solution is to take a (weighted) average price of energy from all the countries or assume a notional price of energy. The advantage of a tracing methodology is that it can be used to allocate payments, rather than MWs, to individual TSOs. This could be achieved by multiplying the losses allocated to imports/exports within each country by the unit price of energy in that country and by multiplying the losses in tie-lines by the average price of energy between the connected countries.

To understand how such mechanism would work assume that the price of energy in the three countries in Fig. 3 is 10, 20, and 30 Euro/MWh, respectively, and that split of charges between the net imports and net exports is 50:50. Then each element of vectors $\Delta\mathbf{P}_{\Sigma}^{(u)}$ and $\Delta\mathbf{P}_{\Sigma}^{(d)}$ is recalculated by multiplying the actual loss by the price of energy in each country or, for tie-lines, by the average price of energy in the two neighboring countries. Referring to Fig. 3(c), the monetary value of the loss in the equivalent link between A and B is

$$\begin{aligned}(151-150)*10+(150-149)*(10+20)/2&\\ +(149-152)*20=-35.&\end{aligned}$$

Similarly, the value of the loss in the equivalent links B-C and A-C is

$$\begin{aligned}&(51-49)*20+(49-48)*(20+30)/2\\ &\quad+(48-47)*30=95 \text{ and}\\ &(52-50)*10+(50-49)*(10+30)/2\\ &\quad+(49-48)*30=70\end{aligned}$$

respectively. Then the relevant vectors are

$$\begin{aligned}\mathbf{P}_{\Sigma}^{(u)}&=[0\quad -35\quad (70+95)]^{T},\\ \mathbf{P}_{\Sigma}^{(d)}&=[(70-35)\quad 95\quad 0]^{T}\end{aligned}$$

and solving (3) and (8) yields

$$\Delta\mathbf{P}^{(u)}=[0\quad -35\quad 153.2566]^{T},\quad \Delta\mathbf{P}^{(d)}=[130\quad 95\quad 0]^{T}.$$

Finally the payment made by the net export in A is $130/2=65$ Euro/hour while the payments made by the net imports in B and C are $-35*(101/152)/2=-11.63$ Euro/hour and $153.2566*(95/95)/2=76.63$ Euro/hour.

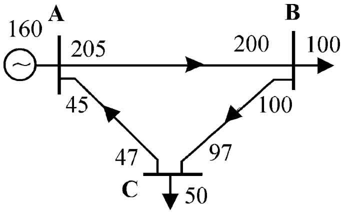

Fig. 4. Example of cyclic flows.

V. CYCLIC FLOWS

One of the problems associated with the implementation of the tracing algorithm is a possible existence of cyclic (also called circular or carousel) flows which make the directed graph of flows to contain cycles [13]. Sometimes cyclic flows are referred to as loopflows [13] but arguably this is confusing as the term loopflows is often used to describe flows which are not on the main contract paths between exporting and importing countries.

When a full nodal network model is used, i.e., without combining internal networks into supernodes, cycles in the digraph may appear due to the presence of phase shifting devices [13]. When the internal networks are combined into supernodes, as proposed here, cycles are quite common due to unbalanced geographical location of generators and loads within each country. An important advantage of the linear equations-based, as opposed to the graph-based, approach to tracing is that cyclic flows are dealt with without any modifications.

Fig. 4 shows a simple example of cyclic flows in a network to illustrate this. The total transmission loss is 10 MW which is assumed to be shared 50:50 between the net exports and imports. Applying the tracing algorithm would give the following result:

$$\begin{gathered}\mathbf{P}_{\Sigma}^{(u)}=[2\ 5\ 3]^{\mathrm{T}},\quad \mathbf{P}_{\Sigma}^{(d)}=[5\ 3\ 2]^{T},\quad \mathbf{P}=[205\ 200\ 97]^{\mathrm{T}}\\ \mathbf{A}_u=\begin{bmatrix}1 & 0 & \frac{-47}{97}\\ -1 & 1 & 0\\ 0 & \frac{-100}{200} & 1\end{bmatrix}\quad \mathbf{A}_d=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ \frac{-45}{205} & 0 & 1\end{bmatrix}.\end{gathered}\tag{10}$$

Solving (3) and (8) yields

$$\begin{aligned}\Delta\mathbf{P}^{(u)}&=[6.1565\ 11.1565\ 8.5782]^{\mathrm{T}}\\ \Delta\mathbf{P}^{(d)}&=[12.8125\ 7.8125\ 4.8125]^{\mathrm{T}}.\end{aligned}$$

The loss allocated to export A is equal to $12.8125*(160/205)/2=5$ MW, which is obviously all the loss allocated to the generators as there are no other net exporters in the system. Similarly, the loss allocated to imports B and C are $11.1565*(100/200)/2=2.789$ MW and $8.5782*(50/97)/2=2.211$ MW, respectively. This shows that the linear-equation based algorithm readily resolves cyclic flows.

By comparison, the graph-based algorithm would fail as it depends on the sequence of calculations starting from the "head" of the graph and finishing in the "tail." The cyclic graph has no "head" and no "tail" so sequential calculations are not possible.

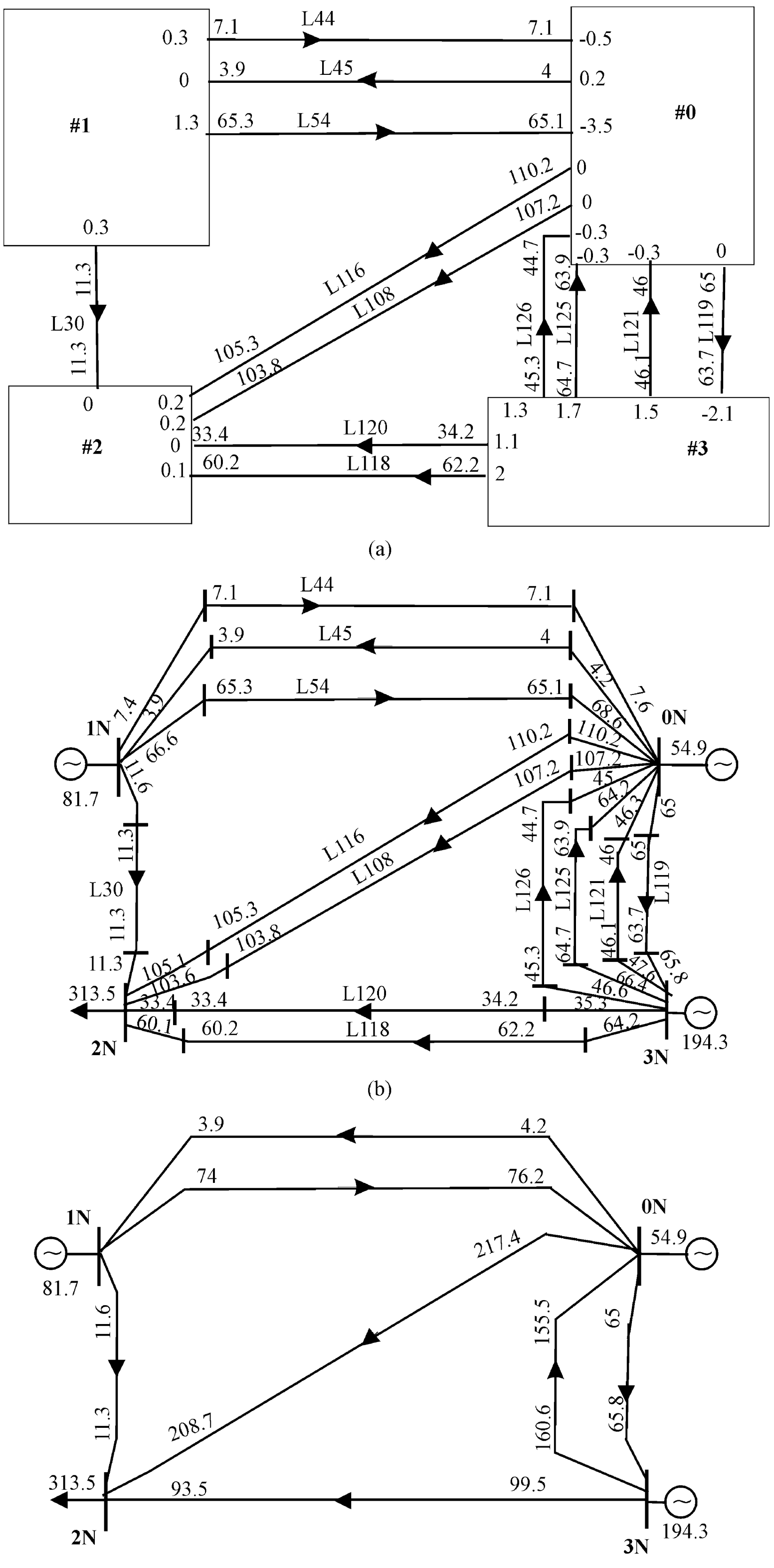

Fig. 5. Equivalent diagram of the four areas with tie-lines: (a) actual flows; (b) equivalent directed graph for tracing; and (c) simplified directed graph.

VI. EXAMPLE

Operation of the proposed loss allocation methodology will now be demonstrated using the IEEE 118 node test system. The aim of the presented example is to show how the proposed methodology works when each control area uses a different internal loss allocation methodology. Although it would be advantageous to test the methodology using real data this proved to be impossible. The authors have contacted a number of utilities in Europe but the utilities were unwilling to provide required net-

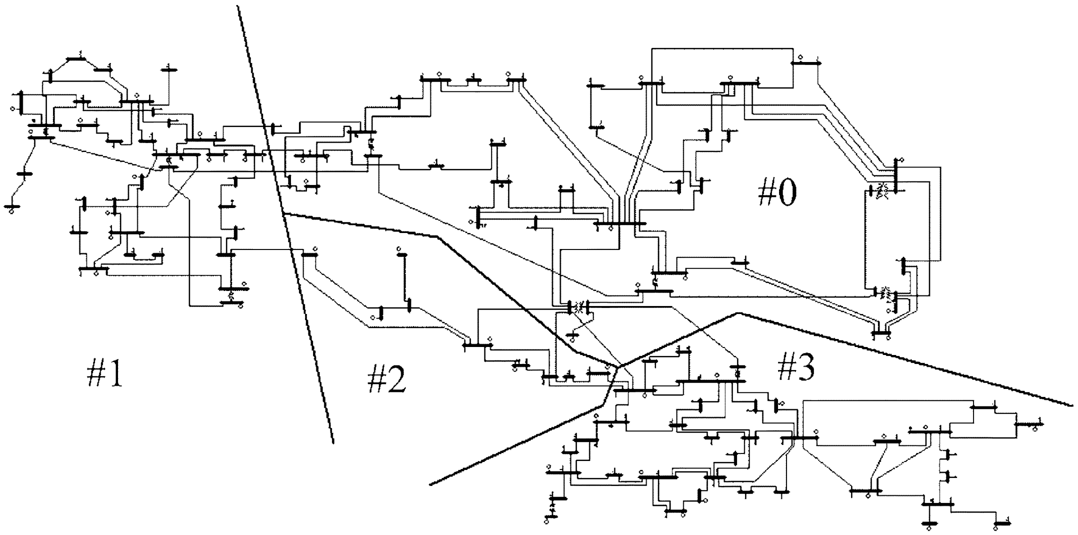

Fig. 6. IEEE 118 node system divided into four areas.

TABLE II
LOSS ALLOCATION IN IEEE 118 NODE NETWORK

	#0	#1	#2	#3	Σ
Internal allocation methodology	marginal	tracing	postage stamp	scaled-down marginal	
Net export	59.6	79.8	-314	188.8	14.2
Loss compensation	-4.7	1.9	0.5	5.5	3.2
Net injection	54.9	81.7	-313.5	194.3	17.4
Loss charge	1.2	0.7	8.7	6.8	17.4
Net charge	5.9	-1.2	8.2	1.3	14.2

work information due to commercial sensitivity concerns, hence underlying the arguments expressed in Section III-A.

Fig. 6 shows the IEEE 118 node network divided into four areas: #0, #1, #2, and #3. The division is completely arbitrary as it aims just to show how the methodology works. In practice, the borders between utilities or countries are well defined. Because of lack of space the IEEE network diagram has been reduced but the full diagram, together with data files, can be downloaded from [14]. Each of the areas is assumed to represent a separate country trading with other countries. Areas #0, #1 and #3 are net exporters while area #2 is a net importer.

To prove that the methodology can provide a unifying framework when each country uses a different internal transmission pricing methodology, it was asserted that area #0 follows marginal charging, area #1 follows tracing, area #2 follows postage stamp (equal pro-rata charges) while area #3 follows a scaled-down marginal loss allocation such that the sum of charges recovers the actual losses incurred in the area (un-scaled marginal charging recovers about double actual transmission losses).

Fig. 5(a) shows a schematic of the four areas with tie-lines and corresponds to Fig. 3(a), Section III. Losses allocated to individual imports/exports within each area are shown next to the associated tie-line. For example the loss allocated to the export from area #1 using line L44 [the top line in Fig. 5(a)] is 0.3 MW while the loss allocated to the import to area #0 using line L44 is -0.5 MW. This loss is negative because area #0 uses the marginal loss allocation and the import from line L44 causes a counterflow. There are more negative loss allocations in areas #0 and #3 due to using internal marginal loss allocation.

Fig. 5(b) shows the digraph used to trace the flow of power due to cross-border trades and corresponds to Fig. 3(b). Loss allocations have been added or subtracted from the flows in tie-lines. Fig. 5(c) shows a simplified digraph in which series-connected sections and parallel lines transferring power in the same direction have been combined. Note that parallel lines with flows in the opposite directions cannot be combined as this would result in a different loss allocation.

Table II shows the results on the basis that the losses are to be allocated at 50:50 ratio between the net imports and the net exports in each area. It has the same structure as Table I. The total loss to be allocated is 17.4 MW which consists of 14.2 MW of actual losses in tie-lines shown in Fig. 5(a) and 3.2 MW of compensations for losses due to cross-border trades. The compensations are equal to the sum of internal loss allocations within each country to individual imports and exports. For example the compensation for area #1 is $0.3 + 0 + 1.3 + 0.3 = 1.9$. The penultimate row shows loss charges due to cross-border trades calculated using tracing methodology. As area #2 is the only net importer, it has been allocated all the losses associated with the imports (50% of $17.4 = 8.7$). All the other areas are net exporters so they share the losses due to cross-border trades. The last row shows net charges equal to the differences between the loss charges and compensations. The sum of net charges gives the sum of actual losses in tie-lines.

Note that there are some cyclic flows in the network. For example the flow in the middle tie-line L45 linking areas #0 and #1 is in the opposite direction than the flows in the remaining tie-lines. A similar situation occurs between areas #0 and #3 where the flow of the right tie-line L119 is in the opposite direction to the flows in the remaining three tie-lines. The linear-equations based tracing algorithm easily resolved these cyclic flows.

VII. CONCLUSION

In this paper, a unifying tracing-based methodology of transmission pricing for cross-border trades has been proposed and validated. It can be interpreted as establishing physical flow-based paths linking net exporting and net importing countries so that compensation payments for losses allocated between the TSOs can be calculated. The methodology allows each country to use a different internal transmission pricing methodology, i.e., it retains the principle of subsidiarity. Countries are represented as supernodes with net imports or exports connected by tie-lines. This prevents allocation of charges to a transit country with balanced internal generation and demand and avoids disclosing commercially sensitive information about internal networks and generation/load profiles. The only data required are the flows in the tie-lines and the charges to individual exports and imports in each country. The methodology is simple, transparent and very fast and it can deal effectively with cyclic flows that make the digraph of flows to become cyclic. The methodology has been illustrated by using loss allocation as an example and tested on the IEEE 118 node network divided into four areas, each with a different internal transmission pricing methodology.

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