IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997

TOPOLOGICAL GENERATION AND LOAD DISTRIBUTION FACTORS FOR SUPPLEMENT CHARGE ALLOCATION IN TRANSMISSION OPEN ACCESS

Janusz Bialek
School of Engineering, University of Durham
Science Laboratories, South Road, Durham DH1 3LE, ENGLAND

Abstract-**This paper introduces a simple novel method of transmission supplement charge allocation based on topological analysis of power flows in the network . The method uses the MW-MILE methodology but analyses the share, not the impact of, individual loads and generators in line flows. This results in positive contributions from all the users hence rescinding the problem of counterflows.**

I. INTRODUCTION

Recent years have seen almost unstoppable pressure towards deregulation and unbundling of the services provided by utilities throughout the world. Among the different components of the electrical supply industry, unbundling of the transmission service is probably the most complicated and difficult as the transmission is, by its nature, a single, highly integrated, electrical business. At the same time, its unbundling is vital for the proper operation of the deregulated industry as "the market power through control of transmission is the single greatest impediment to competition" [1].

This paper is concerned with treating transmission as a separate business of transporting energy from any generator to any area supplier (referred to simply as a load), while maintaining system integrity, with all transactions treated in an equal, non-discriminatory, manner [2]. The problem is how to allocate the total cost of transmission between all the users in an equitable way which at the same time provides them with the correct, market based economical signals. This approach is more general than "wheeling", that is transfer of power between two or more utilities using a transmission network of the third one [3]. Countries like England and Wales, Chile, and many others, are already using Transmission Open Access in practice. Also in the USA, the federal regulation seems to be heading in that direction [1].

Economical theory stipulates that goods and services should be charged on the marginal cost basis. It has been found, however, that the marginal pricing of transmission service is highly volatile, provides perverse economic signals to the transmission company, and fails to recover the total incurred network costs [4,5]. To alleviate this problem, a number of approaches has been proposed. Perhaps the most popular is charging the network users an additional charge on top of the marginal cost charge [5,6]. This charge, which is referred to as the *supplement*, *complementary*, or *revenue reconciliation, charge* may be as high as 70% of the total transmission charge [5].

There is a number of different strategies for supplement charge allocation. Probably the most popular is linking the charge with the actual use of the system by a user. In the *MW-MILE methodology* [7,8,9] the actual use of transmission facilities is expressed, conceptually, by a product of power due to a particular transaction times the distance this power travels in the network. This approach, although based on intuitive rather than theoretical basis, has gained a lot of support as it promotes the maximum use of the existing system [6] and is stable [8], that is provides incentive to the participants to stay in the integrated pool.

A popular approach to MW-MILE method is to use generalised distribution factors to determine the transaction-related power flows [10,9]. This approach is based on the d.c. linearised model of the system and, by using the superposition theorem, allows to determine the impact of a particular load or generator on line flows. Typically the method produces *counterflows*, that is component flows that go in the opposite direction to the total net flow. Rigorous application of MW-MILE method should therefore result in negative charges, that is payments to a participant producing a counterflow. As the transmission service providers feel uneasy about the idea of providing a service and paying the customers who use it, the counterflows may be disregarded [9], taken with the absolute value, or treated in a combined way [8].

This paper proposes a novel, topological, approach to MW-MILE charging which determines the *share*, as opposed to the impact, of a particular generator or a load in every line flow. This topological approach is based on a

96 SM 493-7 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the 1996 IEEE/PES Summer Meeting, July 28 - August 1, 1996, in Denver, Colorado. Manuscript submitted January 2, 1996; made available for printing May 2, 1996.

0885-8950/97/$10.00 © 1996 IEEE

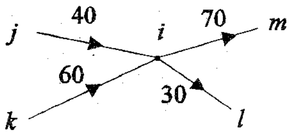

Figure 1 Proportional sharing principle.

recently proposed electricity tracing method [11] and results in positive generation and load distribution factors hence rescinding the problem of counterflows. The method can also be applied to allocate the transmission loss to loads or generators [11,12]. A companion paper describes application of the method to supplement charge allocation due to reactive power flows [13]. In this paper the use of topological distribution factors is demonstrated, and compared with the use of generalised factors, on a simple four-node network.

II. ASSUMPTIONS.

The electricity tracing method [11] is topological in nature, that is it deals with a general transportation problem of how the flows are distributed in a meshed network. The network is assumed to be connected and described by a set of n nodes, m directed links (transmission lines or transformers), $2m$ flows (at both ends of each link) and a number of sources (generators) and sinks (loads) connected to the nodes. Practically the only requirement for the input data is that Kirchhohff's Current Law must be satisfied for all the nodes in the network. In this respect the method is equally applicable to real and reactive power flows and dc currents.

The main principle used to trace the flow of electricity will be that of *proportional sharing*. This is illustrated in Fig. 1 where four lines are connected to node i, two with inflows and two with outflows. The total power flow through the node is $P_i = 40 + 60 = 100$ MW of which 40% is supplied by line j-i and 60% by line k-i. As electricity is indistinguishable and each of the outflows down the line from node i is dependent only on the voltage gradient and impedance of the line, it may be assumed that each MW leaving the node contains the same proportion of the inflows as the total nodal flow P_i. Hence the 70 MW outflowing in line i-m consists of $70\frac{40}{100} = 28$ MW supplied by line j-i and $70\frac{60}{100} = 42$ MW supplied by line k-i. Similarly the 30 MW outflowing in line i-l consists of $30\frac{40}{100} = 12$ MW supplied by line j-i and $30\frac{60}{100} = 18$ MW supplied by line k-i.

The proportional sharing principle basically amounts to assuming that the network node is a perfect "mixer" of incoming flows so that it is impossible to tell which particular inflowing electron goes into which particular outgoing line. This seems to agree with the common sense and with the generally accepted view that electricity is indistinguishable.

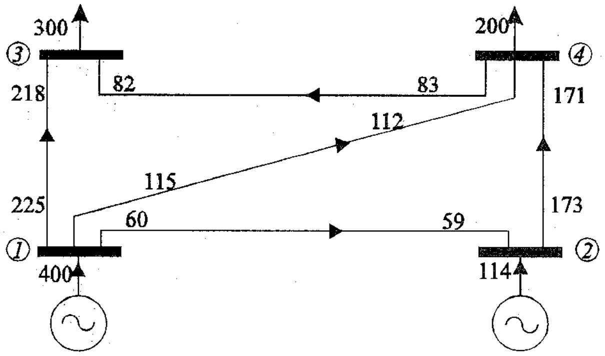

Figure 2 Real power flows in the 4-node network.

As it is impossible to "dye" the incoming flows and check the colour of the outflows, the proportional sharing principle can be neither proved nor disproved. This, however, is irrelevant as the principle will be applied here for financial (or accounting), rather than technical, calculations. In this respect the principle is fair as it treats all the incoming and outflowing flows in the same way. In other words no particular generator or load is distinguished in any way.

III. TOPOLOGICAL ALLOCATION OF THE SUPPLEMENT CHARGE.

The allocation method proposed in this paper is based on the general electricity tracing method [11] and will be illustrated using a working example, Fig. 2, of a simple 4-node network with real power flows. As the electricity tracing method works only when the flows at the beginning and the end of each line are the same, it is necessary to create lossless flows from the lossy flows shown in Fig. 2. One way of achieving this is by introducing additional, fictitious, "line nodes" responsible for the line loss [13] but this results in excessive computational effort for real power flows. A more efficient approach [11] is to break down the total network transmission loss into components to be allocated to individual loads or generators. The *upstream-looking* algorithm will apportion the losses to the loads and allocate the supplement charge to the generators. The *downstream-looking* algorithm will apportion the losses to the generators and allocate the supplement charge to the loads.

A. Upstream-looking algorithm

Assume that it is possible to break down the total transmission loss into components to be added to individual load demands. The sum of the actual demand of a particular load plus the allocated part of the total transmission loss is referred to as the *gross demand.* Obviously the total system gross demand is equal to the total actual generation. Now let us define P_i^g as an unknown *gross nodal power* flow through node i, and P_{ij}^g as an unknown gross line flow in line i-j, both of which would flow if the network was lossless (gross demand equal to actual generation). This would result in a lossless power flow with equal gross flows at the beginning and end of each line. Taking as an example the real power flow shown in Fig. 2, we can find by inspection that $P_1^g = 400$ as there is no line supplying node *1*, $|P_{12}^g| = |P_{21}^g| = 60$, $P_2^g = |P_{12}^g| + P_{G2} = 60 + 114 = 174$, etc.

Generally the gross power balance equation at node i, when looking at the inflows, can be defined as

$$P_i^g = \sum_{j\in\alpha_i^u} \left|P_{ij}^g\right| + P_{Gi} \quad \text{for } i = 1,2,\ldots,n \tag{1}$$

where α_i^u is the set of nodes supplying directly node i (i.e. power must flow towards node i in the relevant lines) and P_{Gi} is the generation in node i. As $\left|P_{ij}^g\right| = \left|P_{ji}^g\right|$, the flow $\left|P_{ij}^g\right|$ can be replaced by $\left(\left|P_{ji}^g\right| / P_j^g\right)P_j^g$. Normally the transmission losses are small and it can be assumed that $\left|P_{ji}^g\right| / P_j^g \cong \left|P_{ji}\right| / P_j$, where P_{ji} is the actual flow from node j in line j-i and P_j is the actual total flow through node j. This corresponds to the assumption that the distribution of gross flows at any node is the same as that of the actual flows. Under this assumption equation (1) can be re-written as

$$P_i^g - \sum_{j\in\alpha_i^u} \frac{\left|P_{ji}\right|}{P_j} P_j^g = P_{Gi} \quad \text{or} \quad \mathbf{A_u P_{gross}} = \mathbf{P_G} \tag{2}$$

where $\mathbf{P_{gross}}$ is the unknown vector of gross nodal flows, $\mathbf{P_G}$ is the vector of nodal generations and $\mathbf{A_u}$ is the *upstream distribution matrix* with its (i,j)-th element equal to

$$\left[\mathbf{A_u}\right]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -\left|P_{ji}\right| / P_j & \text{for } j \in \alpha_i^u \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Note that $\mathbf{A_u}$ is sparse and non-symmetric. If $\mathbf{A_u^{-1}}$ exists then $\mathbf{P_{gross}} = \mathbf{A_u^{-1} P_G}$ and its i-th element is

$$P_i^g = \sum_{k=1}^{n} \left[\mathbf{A_u^{-1}}\right]_{ik} P_{Gk} \quad \text{for } i = 1,2,\ldots,n \tag{4}$$

This equation shows how the i-th gross nodal power is supplied from all the generators in the system. On the other hand, when looking at the nodal outflows, the same P_i^g is equal to the sum of all the gross outflows from node i. Hence the gross outflow in line i-j can be calculated, using the proportional sharing principle, as

$$\begin{aligned} P_{ij}^g &= \frac{P_{ij}^g}{P_i^g} P_i^g = \frac{P_{ij}^g}{P_i^g} \sum_{k=1}^{n} \left[\mathbf{A_u^{-1}}\right]_{ik} P_{Gk} \\ &= \sum_{k=1}^{n} D_{ij,k}^g P_{Gk} \qquad \text{for } j \in \alpha_i^d \end{aligned} \tag{5}$$

where α_i^d is the set of nodes supplied directly from node i and $D_{ij,k}^g = P_{ij}^g \left[\mathbf{A_u^{-1}}\right]_{ik} / P_i^g$. This equation defines $D_{ij,k}^g$ as the *topological generation distribution factor* that is a portion of generation due to k-th generator that flows in line i-j. This definition is similar to that used by Ng to define his *generalised generation distribution factor*s [10]. His method, however, was based on the superposition theorem applied to the dc linearised system model so that his distribution factors represented the *impact* of a particular generation on the line flow which could well be negative. On the other hand the topological distribution factors are based on topological analysis of network flows and represent the *share* of a particular generation in the total line flow. Consequently they are always *positive*.

The total usage of the network, U_{Gk}, by k-th generator can now be calculated assuming that the charge for a particular line will be paid proportionally to the actual use of the line by a given generator. Define the *gross weight* w_{ij}^g of line i-j as a charge per MW due to gross flows, $w_{ij}^g = C_{ij} / P_{ij}^g$ where C_{ij} is the total supplement charge for the use of the line. Now the supplement transmission charge of the k-th generator (or usage U_{Gk}) can be calculated by adding up individual shares (multiplied by line weights) of the generator in all the lines of the system:

$$\begin{aligned} U_{Gk} &= \sum_{i=1}^{n} \sum_{j\in\alpha_i^d} w_{ij}^g D_{ij,k}^g P_{Gk} = \sum_{i=1}^{n} \sum_{j\in\alpha_i^d} \frac{C_{ij}}{P_{ij}^g} \frac{P_{ij}^g}{P_i^g} \left[\mathbf{A_u^{-1}}\right]_{ik} P_{Gk} \\ &= P_{Gk} \sum_{i=1}^{n} \left\{ \frac{\left[\mathbf{A_u^{-1}}\right]_{ik}}{P_i^g} \sum_{j\in\alpha_i^d} C_{ij} \right\} \end{aligned} \tag{6}$$

To determine the charges, it is necessary to invert matrix $\mathbf{A_u}$ and calculate vector $\mathbf{P_{gross}}$ from equation (2).

B. Downstream-looking algorithm

The downstream-looking algorithm allocates the supplement charge to individual loads. The transmission loss is dealt with by breaking it down into components to be subtracted from individual generators. The actual generation of a particular generator minus the allocated part of the total transmission loss will be referred to as the *net generation.* Now let us define P_i^n as an unknown *net nodal flow* through node *i*, and P_{ij}^n as an unknown *net line flow* in line *i-j*, both of which would flow if the network was lossless and loaded with the actual load demand. Under this assumption the total net generation is equal to the total actual demand and a lossless power flow is obtained with equal net flows at the beginning and the end of each line. Inspection of the network shown in Fig. 2 shows that $P_3^n = 300$, as there is no line outflows from node *3*, $\left|P_{3-4}^n\right| = \left|P_{4-3}^n\right| = 82$ and P_4^n = $P_{L4} + \left|P_{3-4}^n\right| = 200 + 82 = 282$, etc.

The net power balance equation at node *i*, when looking at the outflows, can be defined as

$$P_i^n = \sum_{j \in \alpha_i^d} \left|P_{ij}^n\right| + P_{Li} \tag{7}$$

where α_i^d is the set of nodes supplied directly from node *i*. As $\left|P_{ij}^n\right| = \left|P_{ji}^n\right|$ and the transmission losses are small, it can be again assumed that $\left|P_{ij}^n\right| = \left|P_{ji}^n\right| = \left(\left|P_{ji}^n\right| / P_j^n\right) P_j^n$ $\cong \left(\left|P_{ji}\right| / P_j\right) P_j^n$. Now equation (7) can be re-written as

$$P_i^n - \sum_{j \in \alpha_i^d} \frac{\left|P_{ji}\right|}{P_j} P_j^n = P_{Li} \quad \text{or} \quad \mathbf{A_d P_{net}} = \mathbf{P_L} \tag{8}$$

where $\mathbf{P_{net}}$ is the unknown vector of net nodal flows, $\mathbf{P_L}$ is the vector of nodal load demands and $\mathbf{A_d}$ is the $(n \times n)$ *downstream distribution matrix* with its (*i,j*) element equal to

$$\left[\mathbf{A_d}\right]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -\left|P_{ji}\right| / P_j & \text{for } j \in \alpha_i^d \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Note that $\mathbf{A_d}$ is also sparse and non-symmetric. Adding $\mathbf{A_u}$ and $\mathbf{A_d}$ gives a symmetric matrix which has the same structure as the nodal admittance matrix. If $\mathbf{A_d^{-1}}$ exists then $\mathbf{P_{net}} = \mathbf{A_d^{-1} P_L}$ and its *i*-th element is equal to

$$P_i^n = \sum_{k=1}^{n} \left[\mathbf{A_d^{-1}}\right]_{ik} P_{Lk} \tag{10}$$

This equation shows how the *i*-th net nodal power is distributed between all the loads in the system. On the other hand the same P_i^n is equal to the sum of all the net inflows entering node *i*. Hence the net inflow from line *i-j* can be calculated, using the proportional sharing principle, as

$$\begin{aligned} P_{ij}^n &= \frac{P_{ij}^n}{P_i^n} P_i^n = \frac{P_{ij}^n}{P_i^n} \sum_{k=1}^{n} \left[\mathbf{A_d^{-1}}\right]_{ik} P_{Lk} \\ &= \sum_{k=1}^{n} D_{ij,k}^n P_{Lk} \qquad \text{for } j \in \alpha_i^u \end{aligned} \tag{11}$$

where $D_{ij,k}^n = P_{ij}^n \left[\mathbf{A_d^{-1}}\right]_{ik} \Big/ P_i^n$. This equation determines $D_{ij,k}^n$ as the *topological load distribution factor* that is the portion of *k*-th load demand that flows in line *i-j*. This definition is again similar to that of the *generalised load distribution factor* [9]. However the topological factor represents the *share* (which is always positive) of the load in a line flow while the generalised factor determines the *impact* of the load on a line flow and may be negative.

Now define the *net weight* w_{ij}^n as a charge per MW due to the net flow in line *i-j*, that is $w_{ij}^n = C_{ij} / P_{ij}^n$. The total usage of the network, U_{Lk}, by *k*-th load can now be calculated by adding up individual shares of the load (multiplied by line weights) in all the lines in the system:

$$\begin{aligned} U_{Lk} &= \sum_{i=1}^{n} \sum_{j \in \alpha_i^u} w_{ij}^n D_{ij,k}^n P_{Lk} = \sum_{i=1}^{n} \sum_{j \in \alpha_i^u} \frac{C_{ij}}{P_{ij}^n} \frac{P_{ij}^n}{P_i^n} \left[\mathbf{A_d^{-1}}\right]_{ik} P_{Lk} \\ &= P_{Lk} \sum_{i=1}^{n} \left\{ \frac{\left[\mathbf{A_d^{-1}}\right]_{ik}}{P_i^n} \sum_{j \in \alpha_i^u} C_{ij} \right\} \end{aligned} \tag{12}$$

To determine the charges, it is necessary to invert matrix $\mathbf{A_d}$ and calculate vector $\mathbf{P_{net}}$ from equation (9).

IV. NUMERICAL EXAMPLE

Let us apply the algorithm to the real power flow shown in Fig. 2. Due to lack of space, only the results obtained using the upstream-looking algorithm will be shown. The values of line reactances and resistances (in ohms) are shown in Table 1. To simplify the calculations, the charge for the use of a particular line (in per-unit) was assumed to be equal to the line resistance. The total supplement charge was therefore equal to 39.7.

TABLE I LINE RESISTANCES AND REACTANCES

Line	1-2	1-3	1-4	2-4	4-3
$R_{ij} = C_{ij}$	12.75	6	11.7	3.5	5.75
X_{ij}	97	69.5	96	30.8	58

Applying equation (2) gives:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -60/400 & 1 & 0 & 0 \\ -225/400 & 0 & 1 & -83/283 \\ -115/400 & -173/173 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1^g \\ P_2^g \\ P_3^g \\ P_4^g \end{bmatrix} = \begin{bmatrix} P_{G1} = 400 \\ P_{G2} = 114 \\ 0 \\ 0 \end{bmatrix}$$

Inverting the matrix yields

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0.15} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ 0.6908 & 0.2933 & 1 & 0.2933 \\ \mathbf{0.4375} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} 400 \\ 114 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_1^g = 400 \\ P_2^g = 174 \\ P_3^g = 309.76 \\ P_4^g = 289 \end{bmatrix}$$

This result confirms the calculated earlier values of the gross nodal powers. The inverted matrix allows to calculate the topological generation distribution factors using equation (5) and assuming that $P_{ij}^g / P_i^g \cong P_{ij} / P_i$. Table II shows comparison between the topological factors and the generalised factors calculated as in [10]. The topological factors are all positive while some of the generalised factors are negative.

TABLE II TOPOLOGICAL AND GENERALISED GENERATION DISTRIBUTION FACTORS

Line	Topological factors		Generalised factors	
	G1	*G2*	*G1*	*G2*
1-2	0.15	0	0.2197	-0.249
1-3	0.5625	0	0.4815	0.2533
1-4	0.2875	0	0.288	-0.015
2-4	0.15	1	0.2169	0.7479
4-3	0.1268	0.29	0.1099	0.3381

Table III shows comparison between the supplement charges calculated using the topological factors (equation (6)) and the generalised factors, when only positive contributions from generators are taken into account [9]. In this simple example the charges using both methods were quite similar as the negative generalised factors tended to correspond to zero topological factors and both resulted in zero charging. For line *1-3*, however, the topological factor due to *G2* was zero while the generalised one was positive. Inspection of flows in Fig. 2 shows that zero share of generator *G2* in the flow in line *1-3* is quite obvious (*G2* cannot possibly supply this line) while the non-zero generalised factor is much harder to explain.

TABLE III CHARGES BASED ON TOPOLOGICAL AND GENERALISED GENERATION DISTRIBUTION FACTORS

Line	Topological		Generalised	
	G1	*G2*	*G1*	*G2*
1-2	12.75	0	12.75	0
1-3	6	0	5.22	0.78
1-4	11.7	0	11.7	0
2-4	1.21	2.29	1.77	1.73
4-3	3.48	2.27	3.06	2.69
Total	35.14	4.56	34.5	5.2

V. DISCUSSION

The presented method allows to allocate the supplement charge for transmission services to individual loads and/or generators by analysing the topology of network flows. In the author's opinion, the main advantage of the method is in its simplicity. Increasingly, utilities are being run by people with other than engineering background and it is important that a charging method can be understood by anyone with just basic mathematical training.

The physical meaning of topological factors is obvious as they represent a positive share of a particular generator/load in the line flow. For a simple network, as that shown in Fig.2, the factors can be determined by a simple inspection of the network flows. On the other hand the generalised distribution factors use the d.c. linearised network model and they represent the *impact* of the load or generator on a given line flow and may well be negative (so-called *counterflows*). Their use, even for a simple network, is more complicated and cannot be verified by inspection.

Comparison between the charges based on the use of topological and generalised distribution factors, performed on a simple system, showed that the results were broadly similar. Negative generalised factors tended to correspond to zero topological factors and therefore both would give zero contributions. There were, however, cases when the topological factor was zero (no payment) while the generalised factor was positive (non-zero payment). Any assessment of suitability of the topological factors would obviously require a more comprehensive study, using variety of systems and loading situations.

The method requires a base load flow and therefore may be used ex ante to determine the charge allocation assuming e.g. the maximum system loading conditions. Alternatively, the method may be employed ex post, using the actual network flows obtained from the state estimation program, to determine the actual network use over a certain period. There is no need to know the slack node in the network.

The influence of the losses is either inherently included in the allocation to the generators when using the upstream-looking algorithm, or removed from the network when considering the downstream-looking allocation to the loads. Should, however, be necessary to determine the network usage with losses allocated to the loads, the algorithm can easily be modified by considering the gross, instead of the actual, load demands [11].

The method requires inverting an $(n \times n)$ sparse distribution matrix which may be cumbersome for large systems. It should be noted however that only knowledge of the columns corresponding to the generator nodes (upstream-looking algorithm), or load nodes (downstream-looking algorithm), is required enabling the use of sparse inversion methods.

VI. CONCLUSIONS

It is necessary to charge the network users a supplement, or revenue reconciliation, charge as the pure marginal cost pricing fails to recover the total incurred network cost. The standard MW-MILE methodology, based on the dc linearised system model, results in counterflows which have either to be disregarded or taken with the positive sign.

This paper has proposed a novel method of supplement charge allocation based on a recently proposed electricity tracing method. The method allows to trace where the output of every generator go, or input to every load comes from, assuming that nodal inflows are shared proportionally between the outflows. The resulting topological distribution factors allow to determine the network usage by any load and/or generator by summing up the shares of each load/generator in every line flow. As the shares are always positive, no counterflows are encountered and all the charges to the network users are positive.

The method is conceptually very simple and requires inverting a sparse matrix of the rank equal to the number of network nodes. Comparison between the topological and generalised distribution factors, performed for a simple system, showed that although the charge allocation tended to be broadly similar, the topological factors have a physical meaning which can be more readily grasped.

VII. REFERENCES

1. Federal Energy Regulatory Commission "Promoting Wholesale Competition Through Open Access Non-discriminatory Transmission Services by Public Utilities" Notice of Proposed Rulemaking, March 29, 1995.
2. R.D. Tabors "Transmission System Management and Pricing: New Paradigms and International Comparisons" *IEEE Trans. Power Systems*, Vol. 9, No. 1, February 1994, pp. 206-215.
3. M.C. Caramanis, R.E. Bohn, F.C. Schweppe "The Cost of Wheeling and Optimal Wheeling Rates" *IEEE Trans. Power Systems*, Vol. 1, No. 1, February 1986, pp. 63-73.
4. M.C. Calviou, R.M. Dunnet, P.H. Plumptree, "Charging for Use of a Transmission System by Marginal Cost Methods" *Proc. Power System Computation Conference*, Avignon 1993.
5. I.J. Perez-Arriaga, F.J. Rubio, J.F. Puerta, J. Arculuz, J. Marin, "Marginal Pricing of Transmission Services: An Analysis of Cost Recovery", *IEEE Trans. Power Systems*, Vol. 10, No. 1, February 1995, pp. 546-553.
6. D. Shirmohammadi, C. Rajagopalan, E.R. Alward, C.L. Thomas "Cost of Transmission Transactions: An Introduction" *IEEE Trans. Power Systems*, Vol. 6, No. 4, November 1991, pp. 1546-1560.
7. D. Shirmohammadi, P.R. Gibrik, E.T.K. Law, J.H. Malinowski, R. E. O'Donnel, "Evaluation of Transmission Network Capacity Use for Wheeling Transactions", *IEEE Trans. Power Systems*, Vol. 4, No. 4, October 1989, pp. 1405-1413.
8. J.W.M. Lima, M.V.F. Pereira, J.L.R. Pereira, "An Integrated Framework for Cost Allocation in a Multi-Owned Transmission System", *IEEE Trans. Power Systems*, Vol. 10, No. 2, May 1995, pp. 971-977.
9. H. Rudnick, R. Palma, J.E. Fernandez, "Marginal Pricing and Supplement Cost Allocation in Transmission Open Access", *IEEE Trans. Power Systems*, Vol. 10, No. 2, May 1995, pp. 1125-1142.
10. W.Y. Ng, "Generalized Generation Distribution Factors for Power System Security Evaluations" *IEEE Trans. Power App. Systems*, Vol. PAS-100, No. 3, March 1981, pp. 1001-1005.
11. J. Bialek, "Identification of Source-Sink Connections in Transmission Networks" submitted to *IEE Fourth Conf. Power System Control and Management*, London, 16-18 April, 1996.
12. J. Bialek, "Static Transmission Loss Allocation in Power Networks", submitted to *Power System Computation Conference*, Dresden, 19-23 August 1996.
13. J. Bialek, "Supplement Charge Allocation due to Reactive Power Flows in Transmission Open Access", submitted to *IEEE Power Summer Meeting*, Denver 1996.

Janusz Bialek obtained his M.Sc. (1977) and Ph.D. (1981) degrees in Electrical Engineering from Warsaw University of Technology (Poland), where he worked from 1981 to 1989. He is now a lecturer with School of Engineering, University of Durham, England. His interests are in power system analysis problems with particular attention to network pricing, parallel methods, dynamic simulation and state estimation.

Discussion

E Acha, CR Fuerte-Esquivel and H Ambriz-Pérez (The University of Glasgow, Scotland, UK):

We would like to congratulate Dr Bialek for his very interesting and timely paper on charge allocation in transmission open access. This is a subject of growing importance worldwide. His approach is mathematically sound and provides network transparency; a major asset for cases in which service unbundling is required. We would greatly appreciate Dr Bialek's reply to the following questions. Has the author applied the algorithm to power networks of a realistic size? If this is the case, has the algorithm failed to find a solution for any credible operating condition? For instance, the case of circulating power flows is an interesting condition. How efficient is the algorithm when applied to large power networks?

The discussers have solved the numeric example presented in the paper by applying an alternative methodology[A], namely Power Auditing algorithm (PA), and are happy to report that similar results to those given by Dr Bialek's algorithm have been arrived at. The solution is presented below.

The PA algorithm starts from power flows as given by a load flow solution, a dominion is obtained for each generator of the power network. A dominion is a directed graph consisting of one source and one or more sinks. The graph branches relate to transmission lines and transformers, as well as FACTS series devices. In branches which are common to two or more dominions, proportionality is used to determine the power flow contribution of each dominion to the common branch. Loads are handled likewise. The algorithm is general and can be applied equally to networks of any size.

The directed subgraphs of dominions 1 and 2 are shown in Figures 1 and 2, respectively.

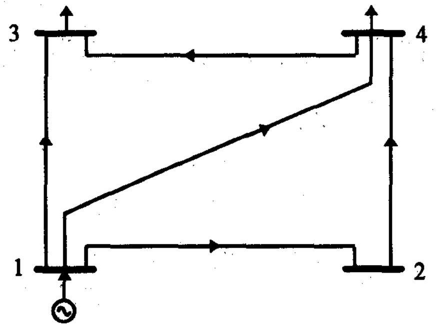

Figure 1. Dominion of generator 1.

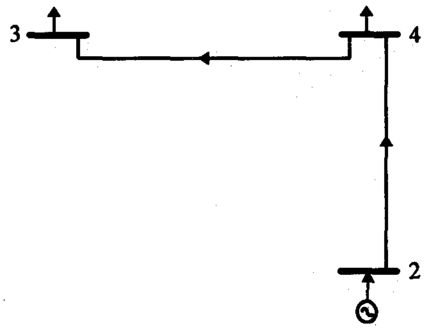

Figure 2. Dominion of generator 2.

In this example there are two common branches. Branch 2-4 and branch 4-3 are both common to dominions 1 and 2. Using the PA algorithm it is straightforward to calculate the contributions of each dominion to common branches 2-4 and 4-3. This information is presented in Table 1.

Table 1. Contribution of dominions 1 and 2 to common branches 2-4 and 4-3.

	Sending end			Receiving end		
Branch	**C^{SR} (%)**	**D_1 (MW)**	**D_2 (MW)**	**C^{RS} (%)**	**D_1 (MW)**	**D_2 (MW)**
2-4	100	59	114	98.84	58.31	112.68
4-3	29.32	49.95	33.04	28.97	49.34	32.64

By way of example, the power flow contribution of dominion 1 at the sending end of transmission line 2-4 is calculated as,

$$C^{2-4} = \frac{173}{59+114} = 1 \text{ and } D_1^{2-4} = 1 \times 59 = 59,$$

as is the contribution of dominion 1 at the receiving end of the same transmission line,

$$C^{4-2} = \frac{171}{59+114} = 0.9884 \text{ and } D_1^{4-2} = 0.9884 \times 59 = 58.31$$

The contributions of dominions 1 and 2 to active power losses in branch 2-4 become readily available from the above result. Power losses and charges for use of line are presented in Table 2.

Table 2. System power losses and charges for use of line.

	Power Loss		Charge for Use of Line		
Branch	**D_1 (MW)**	**D_2 (MW)**	**C^{Ch} (%)**	**Ch_1 (pu)**	**Ch_2 (pu)**
1-2	1	0	1275	12.75	0
1-3	7	0	85.71	6	0
1-4	3	0	390	11.7	0
2-4	0.69	1.32	174.13	1.20	2.30
4-3	0.61	0.40	569	3.47	2.28
Total	12.30	1.72	-----	35.12	4.58

The charges for use of lines 2-4 and 4-3 are calculated as follows:

$$C^{Ch}_{2\text{-}4} = \frac{3.5}{0.69+1.32} = 1.7413$$

$$Ch_1^{2-4} = 1.7413 \times 0.69 = 1.20$$

$$Ch_2^{2-4} = 1.7413 \times 1.32 = 2.30$$

$$C^{Ch}_{4\text{-}3} = \frac{5.75}{0.61+0.40} = 5.69$$

$$Ch_1^{4-3} = 5.69 \times 0.61 = 3.47$$

$$Ch_2^{4-3} = 5.69 \times 0.40 = 2.28$$

It must be noted that the charges based on topological factors (Table III of Dr Bialek's paper) compare well with the charges of Table 2 above.

Once again we would like to compliment Dr Bialek for his very interesting and well written paper.

[A] E Acha, CR Fuerte-Esquivel and H Ambriz-Pérez: 'On the Auditing of Individual Generator Contributions to Active and Reactive Power Flows and Losses in Meshed Power Networks', Submitted to IEE Proceedings, Part C.

Manuscript received September 3, 1996.

D S Kirschen, R N Allan, G Strbac (UMIST, Manchester, UK): The author should be complimented for a timely paper on an important topic. The technique proposed by the author is very similar to that which we have published previously [1]. However, our approach has, we believe, two fundamental advantages over the present one:

- In our paper, we begin with an algorithm which traces the power produced by each generator through a network of arbitrary complexity and hence determines the "domain" of each generator. The intersections between the domains of the various generators defines groups of nodes which are supplied by the same set of generators. We call these groups of nodes "generator commons" and we show that the proportionality assumption which is at the basis of this method is applicable not only to a single node but also to these commons. While each node belongs to one and only one generator common, the number and size of the commons change as the load and generation patterns in the power system change. The state of the power system can be represented by an acyclic graph where the commons are the nodes and bundles of branches connecting these commons are the links. Symmetrical concepts can be developed if we start from the load and work our way back to the generators as in the author's downstream-looking algorithm. We believe that this approach provides a much more systematic and intuitive understanding of the principles of the method.
- Representing the state of the system using an acyclic graph leads to an extremely fast algorithm for computing the contribution of every generator and load to every line flow. This algorithm does not require the inversion of a matrix and is therefore not limited by the invertibility of such a matrix.

[1] D Kirschen, R Allan, G Strbac, "Contributions of Individual Generators to Loads and Flows," paper 96 WM 175-3 PWRS presented at the 1996 IEEE PES Winter Power Meeting.

Manuscript received September 3, 1996.

J. Bialek: The author would like to thank all the discussers for their interest in the paper and their stimulating comments. I am also grateful for pointing to their work I was not aware of when writing my paper. Actually I have recently come across another earlier attempt [X] of using proportional sharing principle for assessing network usage by a particular generator/load based on recursive examination of network nodes, similarly as in the approaches applied by both teams of discussers.

The discussers ask about similarities and differences between their approaches and the method presented. It is important to realise that all three methods are based on the same proportionality principle (or assumption). Therefore it is not surprising that the results obtained by Dr E. Acha compare very well indeed. The only difference between the approaches, apart from organisation of the algorithm which will be dealt with later, lies in the treatment of losses. The algorithm presented by Dr. Acha et al works on actual flows and calculates different distribution factors for the sending and receiving end of each branch while the algorithm presented by Dr. D. Kirschen et al works, as I understand, on average flows. In contrast, the algorithm presented in this paper recalculates all the network flows in order to obtain either fictitious *gross flows* with all the losses added to the flows (upstream-looking algorithm) or equally fictitious *net flows* when all the losses are removed (downstream-looking algorithm). Obviously it is possible to construct an algorithm which, similarly as that proposed by Dr. Kirschen et al, works on average branch flows [Y]. In this case both the upstream and downstream-looking algorithms should give exactly the same result as the algorithm presented by Dr. Kirschen et al. It is worth noting that sum of the losses allocated to generators/loads using either gross or net flows gives the total network loss. On the other hand the allocation of losses using average or actual flows will not add up to the total loss. This can be confirmed by examining Table 2 in the contribution by Dr. Acha et al. The sum of allocated losses is 14.02 MW while the actual total network loss is 14 MW. This discrepancy is small but it may become significant for large networks.

The other difference between the considered methods lies in the organisation of the algorithm. It may seem, as claimed by Dr. Kirschen et al, that the recursive solution is simpler and faster as it does not require inverting a matrix. In fact, the recursive algorithm does require inverting a matrix but without explicitly forming it. To prove this consider Figure 2 and matrix $\mathbf{A}_u$ (shown after Table I) in the paper. Simple re-numbering of network nodes by swapping nodes *3* and *4* gives a triangular matrix which can be solved recursively by forward substitution. Generally, re-numbering of network nodes according to an acyclic directed graph corresponding to network flows (these graphs are shown in Fig. 1 and Fig. 2 in contribution by Dr. Acha et al) will lead to a triangular form of matrix $\mathbf{A}_u$ (or $\mathbf{A}_d$) and therefore to a solution by recursive forward substitution. Therefore the matrix inversion as used in this paper should be treated symbolically and replaced, in actual implementation, by recursive solution of a triangular matrix. The node re-numbering will obviously require analysing "domains" and "commons" of each generator/load, very much like those described by Dr. Kirschen et al, or "dominions" as they are referred to by Dr. Acha et al. This again proves that the approach proposed in this paper can be seen as a formal generalisation of intuitive recursive schemes. The matrix approach, however, allows to use standard linear algebra methods to analyse the existence and computational complexity of solution, ill conditioning etc. The intuitive recursive formulation does not allow to address these very important questions.

Another advantage of the proposed matrix approach is that it allows to deal with circular flows mentioned by Dr. Acha et al. Circular flows, which are often present especially when considering reactive power, create cycles in the directed graph and destroy the triangular structure of the matrix. Recursive schemes must then fail while the proposed method arrives at a solution by solving a linear equation.

Answering other questions asked by Dr. Acha et al, I would like to point out that the example shown was meant to illustrate how the method works and allow easy checks. Currently I am working on application of the method on large networks.

Finally, I would like once again to thank the discussers for their comments and questions which allowed to explain some of the issues not addressed in the paper.

X. "*Transmission Pricing 1993*" Trans Power New Zealand Ltd., February 1993.

Y. J. Bialek "Tracing the flow of electricity" *IEE Proc.-Gen. Transm. Distrib.*, Vol. 143, July 1996, pp. 313-320.

Manuscript received November 6, 1996.