# HIERARCHICAL FREQUENCY GRID MANAGEMENT FOR ARBITRARY RESPONSE FWL IIR FILTER DESIGN USING A GENETIC ALGORITHM

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## ABSTRACT

A new approach of frequency grid management during the design of digital filters is presented. The technique uses a genetic algorithm, for a single-stage design of digital IIR filters with finite wordlength, which performs a two-stage fitness evaluation. The first performed on a coarse frequency grid. Whereas the second being performed on a relatively denser scale. The results obtained indicate a significant reduction in computational overhead.

## 1. INTRODUCTION

During the design of digital filters, the frequency response is often optimised to meet a given specification over a dense grid of frequency points.
The frequency grid points must be dense enough in order to prevent the filter response from violating the given specifications. However, increasing the frequency grid density will consequently increase the computational overhead needed. This is true for most of the techniques developed for designing optimum digital filters in the last two decades. Examples are linear programming [1] and weighted least squares techniques [2]. Only a few researchers have studied the effect of the grid density on digital filter design and all of these have concentrated on the design of FIR digital filters, [3] [4].

In the past few years Genetic Algorithms (GAs) [5] have been applied successfully to a number of design issues regarding both digital and analogue filters [6]. With regard to digital filter design, most of the GA applications have concentrated on the design of FWL FIR filters [7-10], however, recently GAs have also been applied to IIR design problems [11-12] but most of these use the conventional two-stage approach in which the design is produced via an infinite-precession ideal polynomial transfer function which include two stages of approximation.

This paper describes a GA for the design of Infinite Impulse Response (IIR) filters using a cascade of second and/or first order sections. This is an extension of our work in [13]. The GA, in [13], provides a *single-stage* design in that the quantised coefficients are produced directly from the frequency response template. The GA technique described allows the design of digital filters with arbitrary response functions including those which satisfy the classical filter templates. Fitness is evaluated in the frequency range over which a template is specified for a given filter. The template is specified in terms of minimum and maximum limits for a *grid* of frequency values spanning the above range. For each evaluation of frequency response, template violation is checked for and an error value is calculated as the deviation from the minimum/maximum limit being violated. A dense grid is adopted to ensure that local violation of the template by a response is detected. The technique proved successful in satisfying severely restricted templates with relatively small number of coefficient bits.

Although successfully finds solutions the above GA is computationally complex due to the need for a dense frequency grid. Our investigations revealed that significant savings in computation time could be achieved by applying the above fitness function *hierarchically* in two stages. In the first stage, the fitness function evaluates the frequency response of the chromosomes for a given population on a uniform low-density grid which is a low-complexity operation.

This paper commences with a description of the basic GA, highlighting its main features. Next, the concept of hierarchical fitness evaluation is described and justified. Finally, results are given that quantify the computational savings and conclusions are made.

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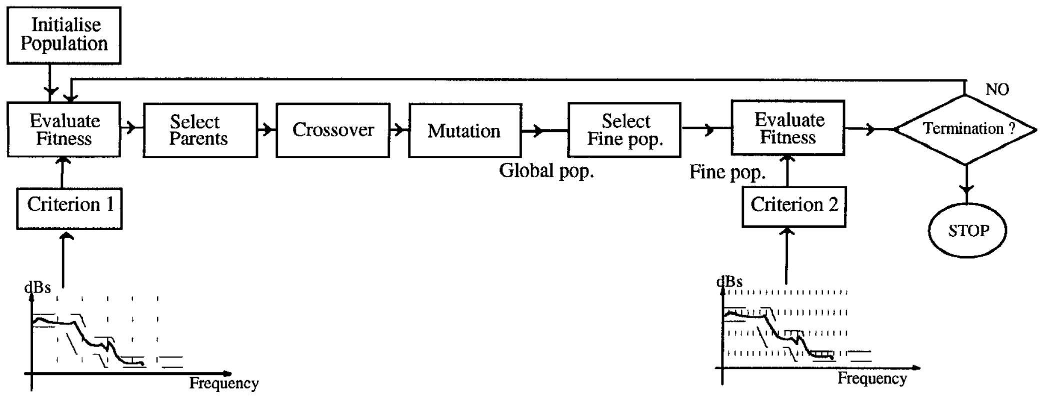
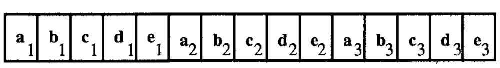



Figure 1: The Genetic Algorithm

## 2. THE GENETIC ALGORITHM

The developed GA is based on a conventional basic form of GA [9]. However, during the genetic evolution the developed GA processes two separate populations. In addition to the original population, termed here *Global population*, used by a conventional GA a group of *N* individuals are selected, subsequent to the formation of a new population, based on their fitness in order to form a relatively smaller population, the *Fine population*. These constitute the *N* fittest individuals in the Global population. The individuals in the Fine population will be subjected to a more comprehensive fitness evaluation the result of which will decide the termination of the evolutionary process. The fitness evaluation procedure will be discussed in more detail in the next section. The evolutionary procedure is illustrated in figure 1.




$a_i$ $b_i$ $c_i$ $d_i$ $e_i$ **: Coefficients of the ith cascade stage**

$i \in \{1,3\}$

Figure 2 : Representation scheme

Each second order filter section has the form :

$$\frac{a + bz^{-1} + cz^{-2}}{1 + dz^{-1} + ez^{-2}}$$

where a,b,c,d, and e are the multiplier coefficients. In the GA genes are represented as a string comprising of sequences of N bit binary numbers (N is the filter wordlength). Each of the binary numbers is associated with one of the multiplier coefficients. A filter structure consisting of M cascaded stages will be formed by concatenating M strings each representing one of the cascade stages. The representation scheme for a filter of three cascaded stages (M=3) is illustrated in figure 2.

The initial population is derived from a set of infinite precision coefficients which are produced using a set of appropriate routines within the Matlab signal processing environment [14]. Next, the above coefficients are rounded to the nearest finite word length number and then transformed to a finite range of values using a random process. For the examples tried, the addition of a uniformly selected random number in the range [-3,+3] was found to be satisfactory in order to breed promising solutions through successive generations to fully correct designs (it could clearly be seen that such designs could not be obtained by simple rounding or truncation). A population size of a fifty was found to provide sufficient genetic diversity. Uniform two-point crossover is employed with a mutation procedure which involves the addition/subtraction of a random number to a random position in the chromosome. The GA performed best at crossover and mutation rates of 90% and 4% respectively.

## 3. FITNESS EVALUATION

Fitness is evaluated in the normalised frequency range [0,1] over a uniform grid of frequency points. A template specification is defined in dBs. A total error is defined (E), which is a frequency-weighted sum-squared error across all frequency grid points, with the error at any frequency being the amount that the magnitude response is outside the template, and zero otherwise. Thus the total magnitude error is zero if it fully satisfies the magnitude template. To avoid divide-by-zero errors as a consequence, some small value, $\varepsilon$ (set at $10^{-5}$), is added to the total magnitude error. The reciprocal of the resulting value was taken as the value for the fitness. Hence, the fitness of the complete response is defined as follows:

$$fitness = \frac{1}{E + \varepsilon}$$

For the Fine population, fitness is evaluated on a relatively fine frequency grid scale in increments of *f*

Hertz. Whereas for the Global population the grid increments are increased to some multiple of $f$, i.e. $(n\times f)$ Hertz, $n$ being an integer value. Our investigations indicated that a good trade-off between accuracy and computational saving is achieved when the values of $f$ and $n$ are one Hertz and five respectively.

## 4. RESULTS

We demonstrate our results by using the GA to design a low pass filter with a passband range of [0,0.56] and stopband region of [0.68,1] with attenuations of 1dB and 80dB respectively. Typically, the GA takes about 30 generations in order to obtain a first satisfactory solution. After another five generations approximately 20% of the population are fully correct designs.

The computational savings which could be achieved using hierarchical fitness evaluation are demonstrated in figures 3-5. In figure 3, the number of individuals in Fine population, i.e. their fitness evaluated on fine grid, are gradually increased and the minimum number of generations taken to reach a first correct design is monitored. As could be seen, with only five individuals in the Fine population forty seven generations are required whereas if the number is increased to forty only twenty generations are required. Figure 4 is a plot of computational complexity per generation, measured as the total number of fitness evaluations performed for a specific Fine-population/Global-population size, with different population sizes in Fine population. Clearly, for only five individuals being evaluated on the fine grid 3000 (50*10+5*500) evaluations per generation are required in comparison with 25000 for that using only the fine grid. Hence from the two figures we can deduce that maximum saving in computational overhead could be achieved with only five individuals being fully evaluated providing a saving of 81.2%. Whereas forty individuals provide a saving of 45.3%. The results in figures 3 and 4 are the average of four runs.

Figure 5 illustrates the number of correct designs in the Fine population as the genetic process evolves. This multiplicity is important since it offers the designer alternative designs to use. Figure 6 shows responses for two typical designs produced by the GA for the filter above. This pattern of computational savings achieved have been confirmed with other filter designs.

## 5. CONCLUSIONS

In this paper a technique is described for efficient frequency grid management during the design of digital filters. A genetic algorithm is used which applies a fitness function hierarchically to members of its successive populations. Significant computational saving has been achieved.

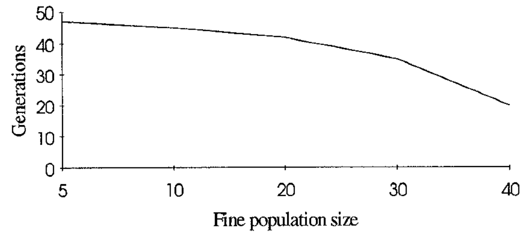



Figure 3: Minimum generations for various Fine population sizes.


Figure 4 : Computational savings with variation in Fine population size.

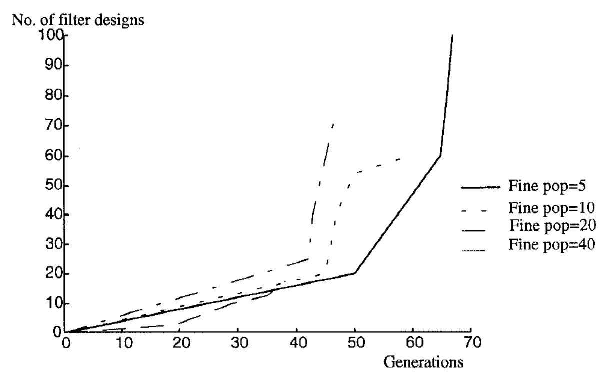



Figure 5: Fine population fitness with different Fine population sizes.

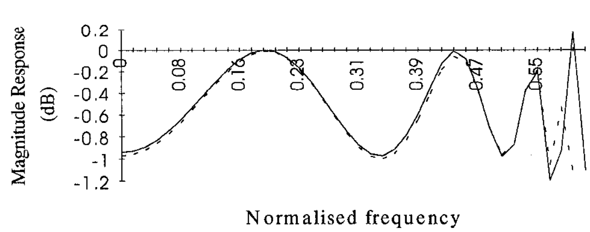



Figure 6: Pass band frequency response for two GA designed filters

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