

Frequency and Scale Domain Analysis of Complex Quadrature Embolic Doppler Ultrasound Signals

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Abstract

Early and accurate detection of emboli is important for monitoring of preventive therapy in stroke-prone patients. One of the problems in detection of emboli is the identification of an embolic signal caused by very small emboli. The amplitude of the embolic signal may be so small that advanced processing methods are required to distinguish these signals from Doppler signals arising from red blood cells. The windowed Fourier transform has been widely used by commercial Doppler ultrasonic systems. An alternative approach is the wavelet transform. Here, we present comparison of time-frequency (based on complex fast Fourier transform) and time-scale (based on complex wavelet transform) analysis. The results reveal that the wavelet transform provides an optimized temporal resolution and time localization for analysis and detection of embolic signals.

1. Introduction

Early and accurate detection of emboli may be important in the identification of patients at high risk of stroke. Embolic signals are short duration transient like signals with high amplitude because of their reflectivity and size compared to that of red blood cells [1][2]. Asymptomatic circulating emboli can be detected by transcranial Doppler ultrasound [3]. The majority of approaches used for automated embolic signal detection have been based on the fast Fourier transform (FFT) processing, which is the standard processing used by Doppler ultrasound machines [4][5]. Embolic signals are highly non-stationary and last only for a short time. Signal analysis methods, which assume that the signal is stationary, are not appropriate. Therefore time-frequency (TF) analysis of embolic signals is necessary. The windowed Fourier transform (WFT), which is an implementation of TF analysis based on the FFT, has been widely used by commercial Doppler ultrasonic systems. However, the FFT is not ideally suited to the analysis of embolic signals due to an inherent trade-off between temporal and frequency resolution [6]. An alternative analysis method is the wavelet transform (WT), which is ideally suited to the analysis of nonstationary transient like signals such as embolic signals [7].

2. Time-frequency Analysis

The WFT introduces time dependency in the Fourier transform by pre-windowing the signal $s(t)$ around a particular time t , and calculating its FFT. This is repeated for each time instant t . The WFT of $s(t)$ is given by

$$F_s(t, f) = \int_{-\infty}^{+\infty} s(\tau) g^*(\tau - t) e^{-j2\pi f\tau} d\tau \quad (1)$$

where $g(t)$ is a short time analysis window function. Because multiplication by relatively short window suppresses the signal outside a neighborhood around analysis time point $\tau=t$, the WFT is a local spectrum of the signal $s(t)$ around a particular t . The WFT results of quadrature Doppler signals are presented as sonogram, which is a form of TF representation and defined as $|F_s(t, f)|^2$. The information concerning flow direction, which is encoded in the phase relationship between in-phase and quadrature-phase channels, is decoded in frequency domain by complex FFT and mapped over the TF plane. If the positive frequency range on a TF plane is formed by the signals resulting from the forward flow, the negative frequency range represents the signals resulting from the reverse flow [8].

3. Time-scale Analysis

The WT decomposes a time series into time-scale (TS) space and enables one to determine both dominant modes of variability and how those modes vary in time. It is performed by projecting a signal $s(t)$ onto a family of zero-mean functions deduced from an elementary function $\psi(t)$ by translations and dilations, and given by

$$W_s(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} s(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (2)$$

where $\psi^*(t)$ is the analyzing wavelet. In the TS analysis, in which the wavelet can be defined as a complex function, the variable a ($\neq 0$), which controls the scale of the wavelet, can be extended to the negative values. The variable b is the time translation and controls the position of the wavelet. The basic difference between the WT and the WFT is that when the scale factor a is changed, the duration and the bandwidth of the wavelet are both changed but its shape

remains the same. The WT uses short windows at high frequencies and long windows at low frequencies in contrast to the FFT, which uses a single analysis window. This partially overcomes the TF resolution limitation of the WFT. The WT results of quadrature Doppler signals are presented as scalogram, which is a form of TS representation and defined as $|W_s(a,b)|^2$. In this representation horizontal axis is the time and vertical axis is the scale. The signal intensity is color coded at the related time-scale coordinate.

Similar to the complex FFT, the use of complex wavelets such as Morlet wavelet [9] or Cauchy wavelet [10] allows negative and positive scales to be utilised for mapping directional wavelet coefficients on TS plane [11][12]. The WT for processing quadrature Doppler signals can be implemented in such a way that the wavelet coefficients produced by the forward flow components are obtained when the scale is positive and the wavelet coefficients produced by the reverse flow components are obtained when the scale is negative. If the Fourier transform of a mother wavelet $\psi(t)$ is defined as $\Psi(\omega) = \Psi^+(\omega) + \Psi^-(\omega)$ (sum of its positive and negative frequency components), in addition to the required standard properties, such a wavelet must also satisfy the following property:

$$\Psi(\omega) = \begin{cases} \Psi^+(\omega) & \text{if } a > 0 \\ \Psi^-(\omega) & \text{if } a < 0 \end{cases} \quad (3)$$

This can be accomplished by the sine-cosine formulation, which naturally exists in some common wavelets such as Morlet wavelet.

4. Methodology

Consecutive quadrature embolic Doppler signals from patients with symptomatic carotid stenosis were used for analysis. Recordings were made from a middle cerebral artery using an axial sample volume of 5 mm. The quadrature ultrasonic Doppler signals had been recorded using a transcranial Doppler system. The sampling frequency was 7150 Hz and the data length was 2048 point (286 ms). The embolic signals were included in the first half of the data. The recorded quadrature ultrasonic Doppler signals were analysed using both the complex FFT and the complex WT. For the WT analysis, a 64 scale WT using complex Morlet wavelet was applied to quadrature signals over positive and negative scales, producing a total of 128x2048 data array representing the TS distribution of the signals.

For investigation of the effect of window size, the FFT was evaluated for each sample point. Prior to the FFT, the Hanning window was applied. The FFT size was fixed to 512 and the parameters were evaluated for six different window sizes (16, 32, 64, 128, 256 and 512). The TF representation of embolic signals using the WFT and the WT was compared by calculating

embolic signal to background blood signal ratio (EBR), half width maximum of the embolic signal power increase in the time domain (HWMT) as an estimate of temporal resolution, half width maximum in the frequency domain (HWMF) as an estimate of frequency resolution, and absolute time of embolic signal onset (ESO) as an estimate of the accuracy of temporal localisation [7]. The EBR is defined as following;

$$EBR = 10 \log \frac{A_{peak}}{B_{avg}} \quad (4)$$

where A_{peak} is the power at frequency with maximum power, and B_{avg} is the average power of the background intensity. This was calculated using the second 1024 data points of the total data.

For the time resolution, the TF/TS distribution of the embolic signal was integrated over all frequencies/scales resulting in the instantaneous power (IP) of the signal. The HWMT was defined as the distance between the two 50% points of maximum of the normalised IP. This parameter indicates the amount of the time spread depending on the window size considered for the WFT and wavelet function considered for the WT. For the frequency/scale resolution, the TF/TS distribution of the embolic signal was integrated over all time resulting in the energy spectrum of the signal. The HWMF was defined as the distance between the two 50% points of maximum of the normalised energy spectrum. This parameter indicates the amount of the frequency spread depending on the window size considered for the WFT and wavelet function considered for the WT. The ESO was defined as the time at which the power increase reached 20% of the normalised IP curve. The ESO indicates how the time localisation properties of the WFT and the WT are influenced by the window size and the wavelet function respectively. These were compared with the ESO values estimated from the IP of quadrature Doppler signals in the time domain.

Table 1. Mean (and standard deviations) of the EBR, HWMT, HWMF, and ESO for the 25 embolic signals, and 2-tail significance values of the localization of the WFT and the WT compared to the time domain signal.

	EBR(dB)	HWMT(ms)	HWMF(Hz)	ESO(ms)
WFT(16p)	12.2(1.7)	6.8(4.5)	1018(245)	65.9(6.2)
WFT(32p)	12.9(1.5)	7.2(4.8)	769(307)	65.5(6.4)
WFT(64p)	13.4(1.2)	8.3(4.6)	623(332)	64.5(6.5)
WFT(128p)	13.1(1.3)	10.9(4.3)	547(368)	60.7(7.4)
WFT(256p)	12.5(1.4)	17.7(3.9)	498(340)	52.6(10.7)
WFT(512p)	11.6(1.4)	38.8(17.6)	497(342)	34.2(16.9)
WT	13.9(1.8)	7.1(5.2)	627(298)	65.9(6.7)
ESO measured from time domain signals (ESOT)				66.2(6.6)
2-tail significance values				
ESOT vs WFT(16p)				0.329
ESOT vs WFT(32p)				0.014
ESOT vs WFT(64p)				0.000
ESOT vs WFT(128p)				0.000
ESOT vs WFT(256p)				0.000
ESOT vs WFT(512p)				0.000
ESOT vs WT				0.129

5. Results and Conclusion

Mean and standard deviations of the EBR, HWMF, HWMT and ESO for the 25 embolic signals are presented in Table 1. Although the differences are insignificant the WT gives the best EBR ratio for this data set. The HWMT for the WFT increases and the HWMF for the WFT decreases with the increasing window size. The HWMT for the WT was as good as the WFT with short window sizes. The HWMF for the WT was as good or better than the WFT with window sizes up to 128 point. Time localization using the WT was also much better than the WFT and shows very close agreement with the measurement from the time domain signal.

The TF description of a signal using the WFT is mainly a function of the analysis window size as shown in Figure 1 and Table 1. In Figure 1, the signal has a complex frequency and amplitude structure. It is apparent that the variations in time are well preserved for shorter window sizes (Figure 1(b)). In contrast, the frequency contents of the signal are well preserved for longer window sizes (Figure 1(g)). A progressive ambiguity in separating amplitudes with increasing effective window size is clearly evident. In contrast, the dominant frequency contents of the signal become more distinctive with increasing window sizes. The results in Table 1 indicates that the mean EBR is lower for shorter windows. It increases as a function of window size up to a certain length (128 point), but decreases again with the longer window sizes. This means that a short window introduces broadening in frequency while a long window introduces broadening in time. These two extreme cases appear to cause a decrease in the EBR ratio by possibly increasing the background level in either the frequency or time space. Although it depends on the actual duration of an embolic signal, between these two extreme cases, medium-sized windows (64 point or 128 point) should be considered as they reflect the TF behavior of an embolic signal most clearly (Figure 1(d) and Figure 1(e)). For the embolic signals, improvement in the frequency resolution was insignificant for window lengths longer than 128 points. Conversely improvement in the time resolution was insignificant for window lengths less than 64 points. In general, short windows give better results for short duration embolic signals and longer windows give better results for longer duration embolic signals [13].

In contrast, the TS description of a signal using the WT is mainly a function of the wavelet used as illustrated in Figures 1 and 2. In Figure 1(h), the TS description of the same signal using Morlet wavelet is comparable to the TF description of the signal using the WFT. The wavelet transform appears well suited to the analysis of embolic signals offering an optimised TF resolution and superior time localisation compared to the WFT as evidenced in Table 1, Figure 1 and Figure 2. This is because the WT is local in both time and

frequency domains. Consistent with the visual assessment, the WT had as high temporal resolution as the WFT with short window and the best EBR as given in Table 1.

Although the WFT has been widely used by commercial Doppler ultrasonic systems, it is not ideally suited to analysis of short duration embolic signals due to an inherent trade-off between time and frequency resolution as demonstrated in this study. An alternative approach is the WT, which is ideally suited to the analysis of non-stationary signals. Here, we presented a comparison of TF and TS analysis. The results reveal that the wavelet transform provides an optimized time/frequency resolution and time localization for analysis and detection of embolic signals.

12. References

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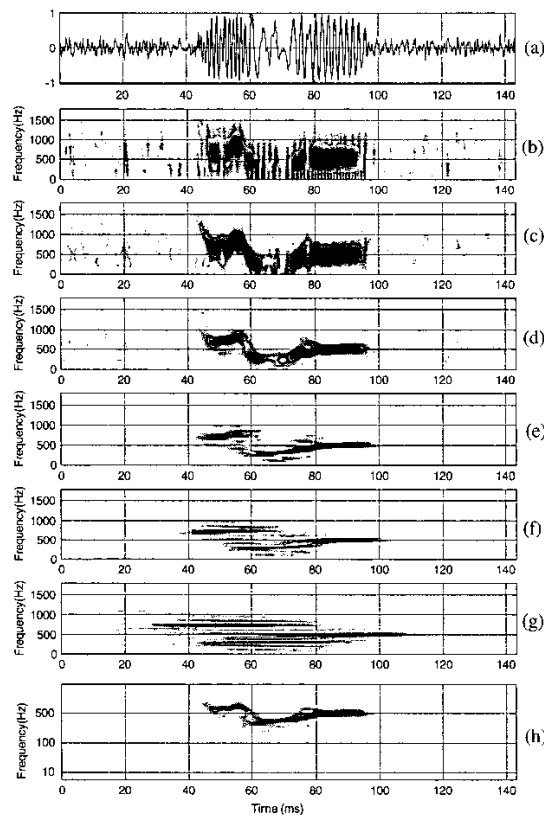


Figure 1 (a) An embolic signal having a complex amplitude and frequency structure (only forward flow), related TF distributions for (b) 16 point, (c) 32 point, (d) 64 point, (e) 128 point, (f) 256 point, (g) 512 point effective window (Hanning) sizes, and (h) TS distribution (64 scales Morlet wavelet). Note that here the vertical axis is given in frequency.

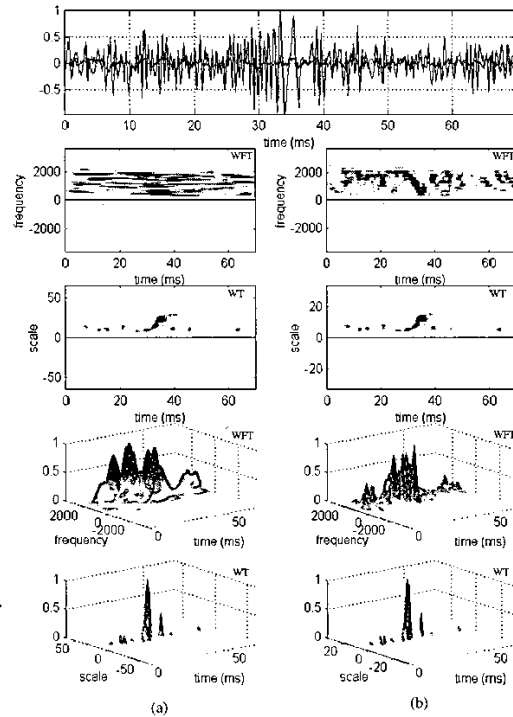


Figure 2 A low intensity embolic signal and corresponding 2d and 3d TF and TS distributions for (a) the 128 point WFT (Hanning window) and the 64 scales WT (complex Morlet wavelet), (b) the 32 point WFT (Hanning window) and the 32 scales WT (complex Morlet wavelet).