Performance Based Engineering with a Bivariate PDF of Fire Size and Vent Opening

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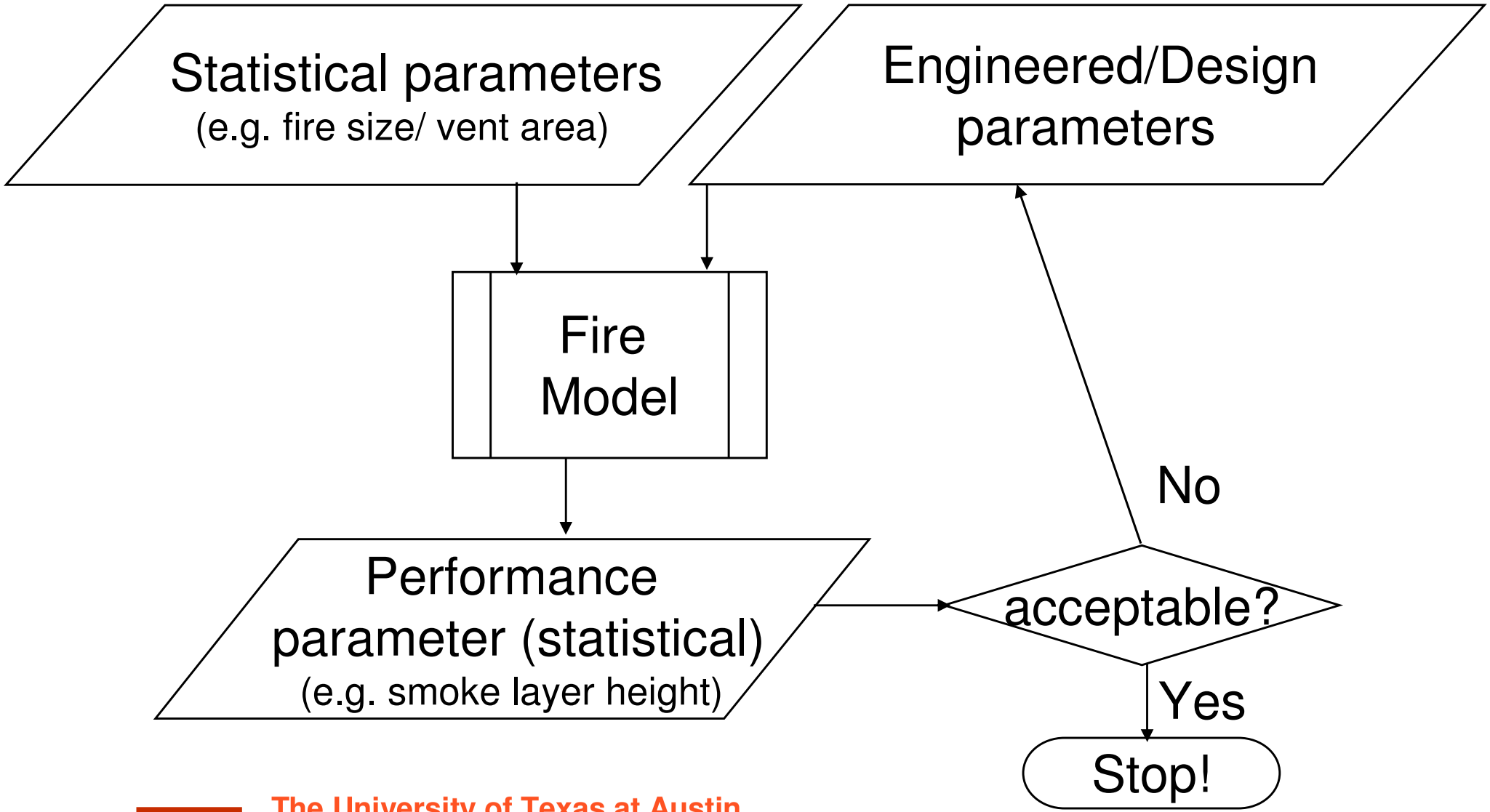
Outline of the Presentation

- Treatment of Uncertainty in Performance Based Fire Design
- Definition of the Problem
- Monte Carlo Simulations
- Quadrature Method of Moments (QMOM) with CDF reconstruction
- Performance evaluation of QMOM
- Conclusions

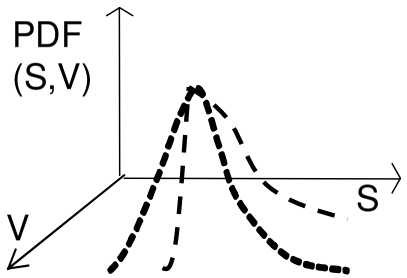
The Role of Uncertainty in Performance Based Fire Design

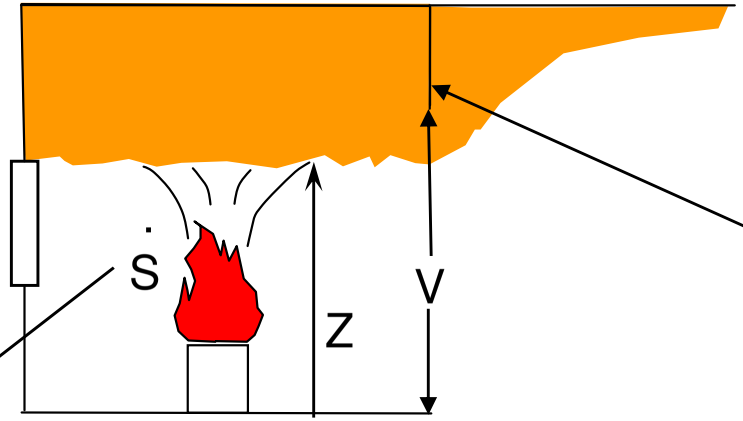
- Performance Based Fire Design (PBFD) requires that design satisfies a performance requirement.
- In designing for a fire scenario, many variables are unknown.
- Based upon history (statistics) one can develop a probability distribution for these unknowns (e.g., fire size/vent opening).
- In practice a fire model can be run for fire inputs selected from the fire size probability distribution.
- A cumulative distribution function can be constructed for the output design parameter (e.g., smoke layer height, time to structural failure, egress time etc.).

Flowchart Representation of PBFD



Schematic of Statistical Propagation of Uncertainty

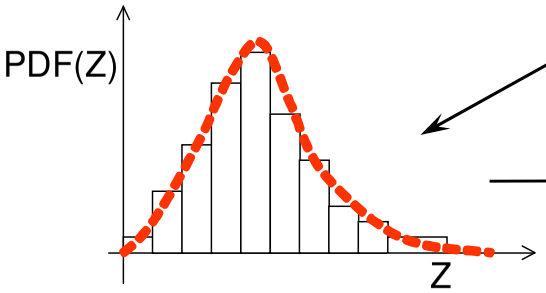




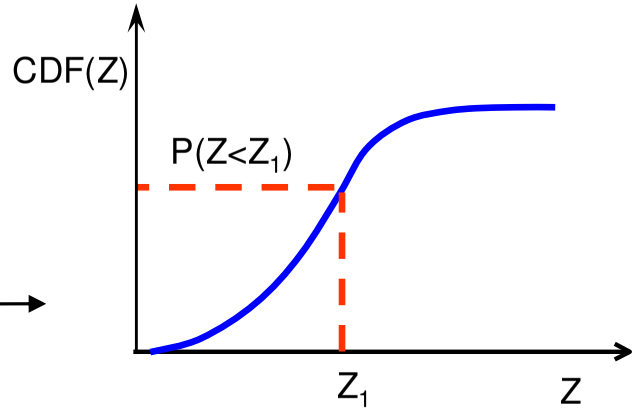
Engineered / design parameters

INPUT: Bivariate PDF of fire heat release rate (S) and vent opening area (V)

Fire Model

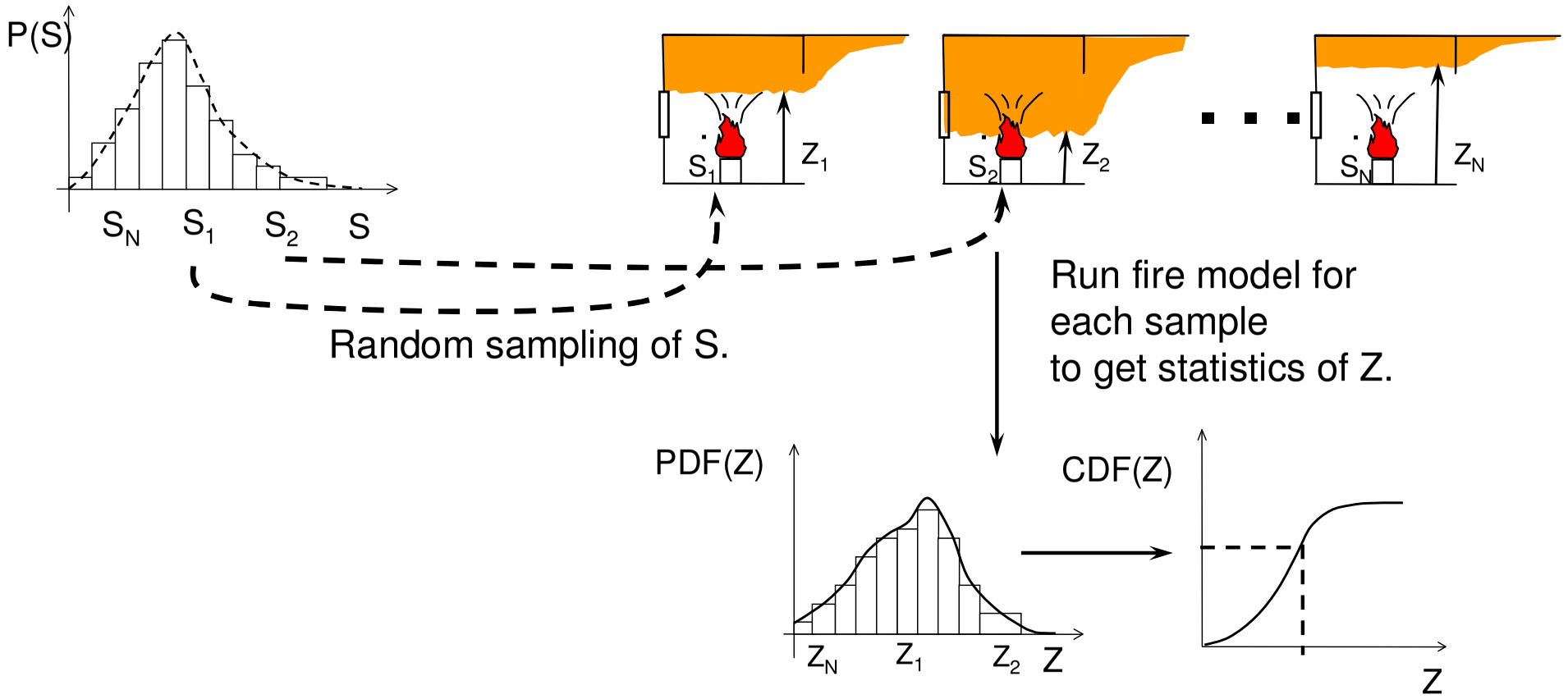


Univariate PDF of performance parameter (e.g. smoke layer height (Z)).



OUTPUT: CDF of performance parameter

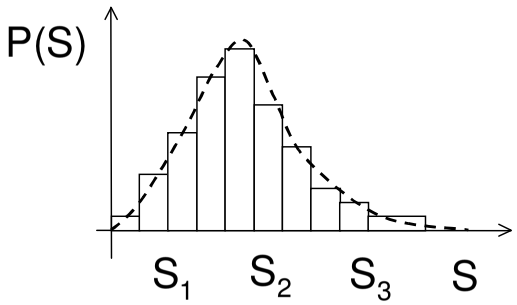
Solution technique: Monte Carlo Simulations*



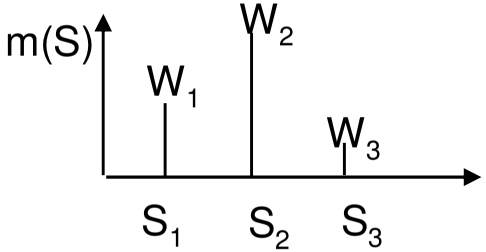
- N is typically large to get good statistics.
- If fire model is complex, then can be very expensive to compute.

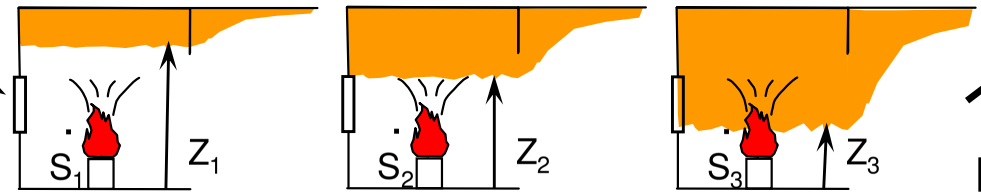
K. Notarianni, “Uncertainty” in SFPE Handbook.

Quadrature Method of Moments (QMOM)*



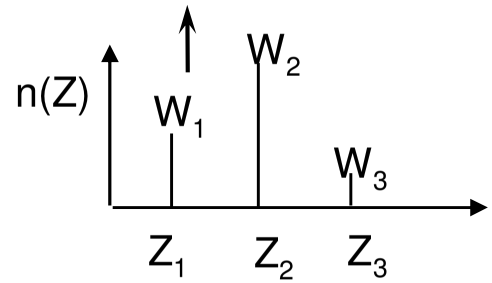
Moments of P(S)



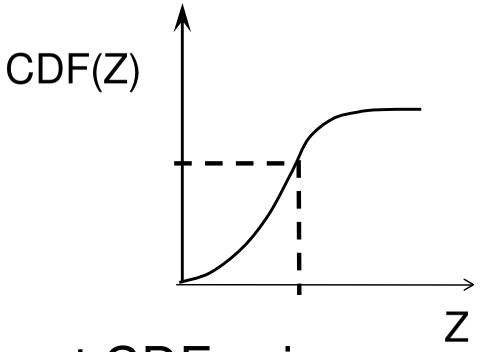


Run fire model for only three samples

Moments of n(Z)



Reconstruct CDF using moment matching

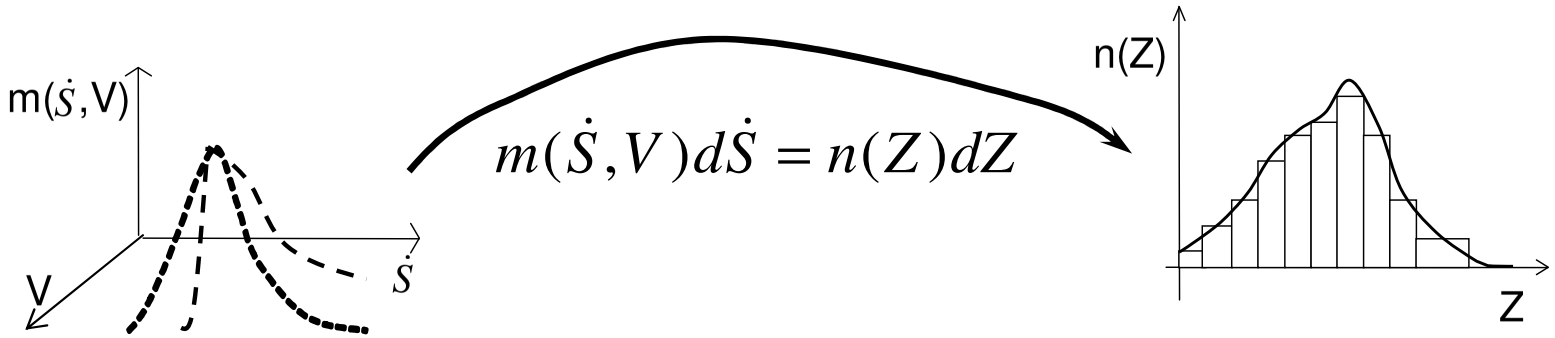


- Typically 2 or 3 samples used.
- Drastic reduction in computational cost.
- Need to check the accuracy of the CDF reconstruction from the moments.

*Upadhyay and Ezekoye (2006). Fire Safety Journal (under review).

Mathematical Formulation of QMOM for the Bivariate Case

A. Mapping of input and output PDFs



B. Fire Model used for the transformation between variables

$$Z(\dot{S},V,t_{cr}) = \int_{t_0}^{t_{cr}} f(\tau,\dot{S}(\tau),V(\tau))d\tau$$

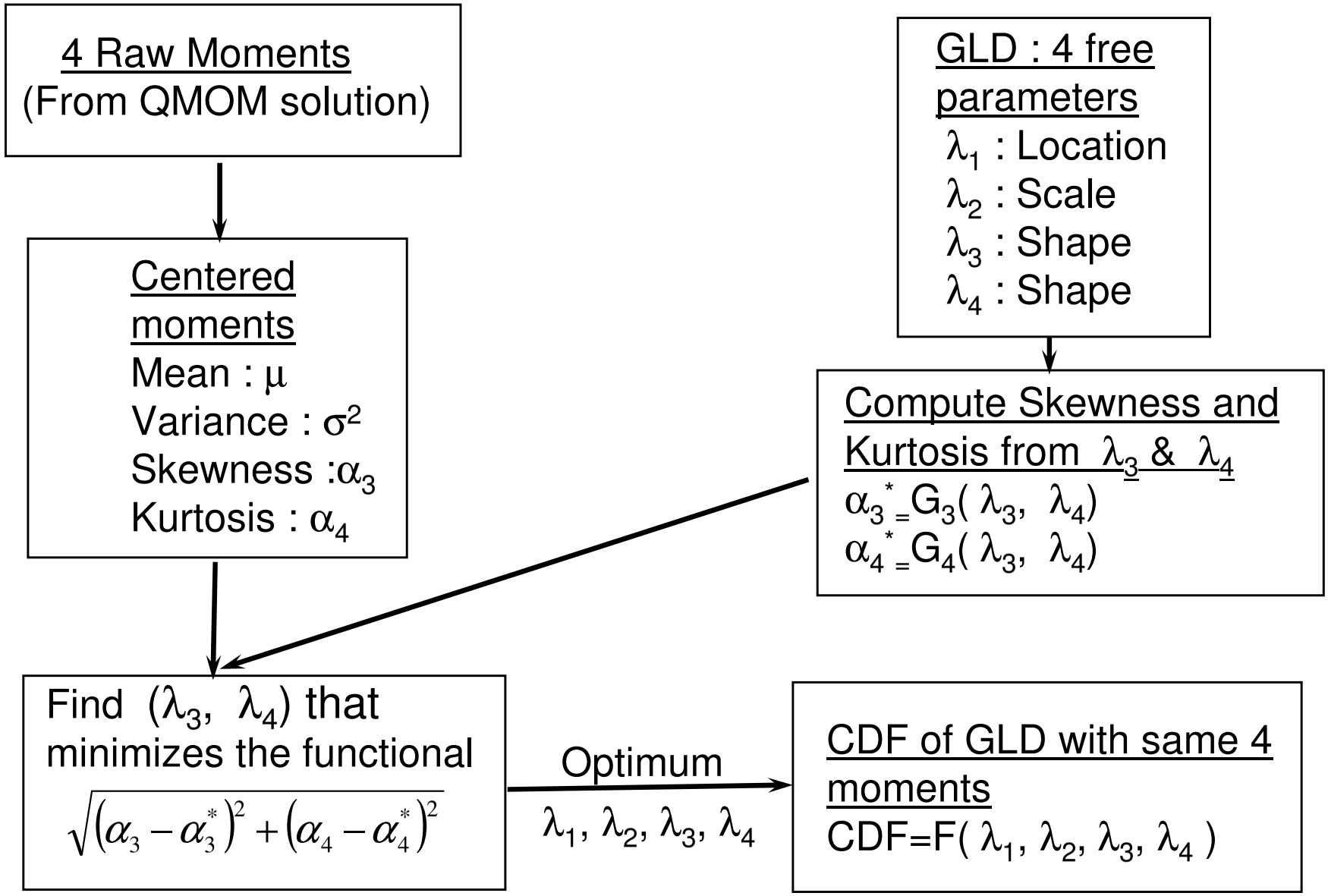
C. Moments of n(Z) from (A) and (B):

$$M_k^{(Z)} = \int_{Z_{\min}}^{Z_{\max}} Z^k n(Z)dZ = \int_{\dot{S}_{\min}}^{\dot{S}_{\max}} \int_{V_{\min}}^{V_{\max}} Z(\dot{S},V)^k m(\dot{S},V)d\dot{S}dV$$

D. Use quadrature to evaluate double integral in (C)

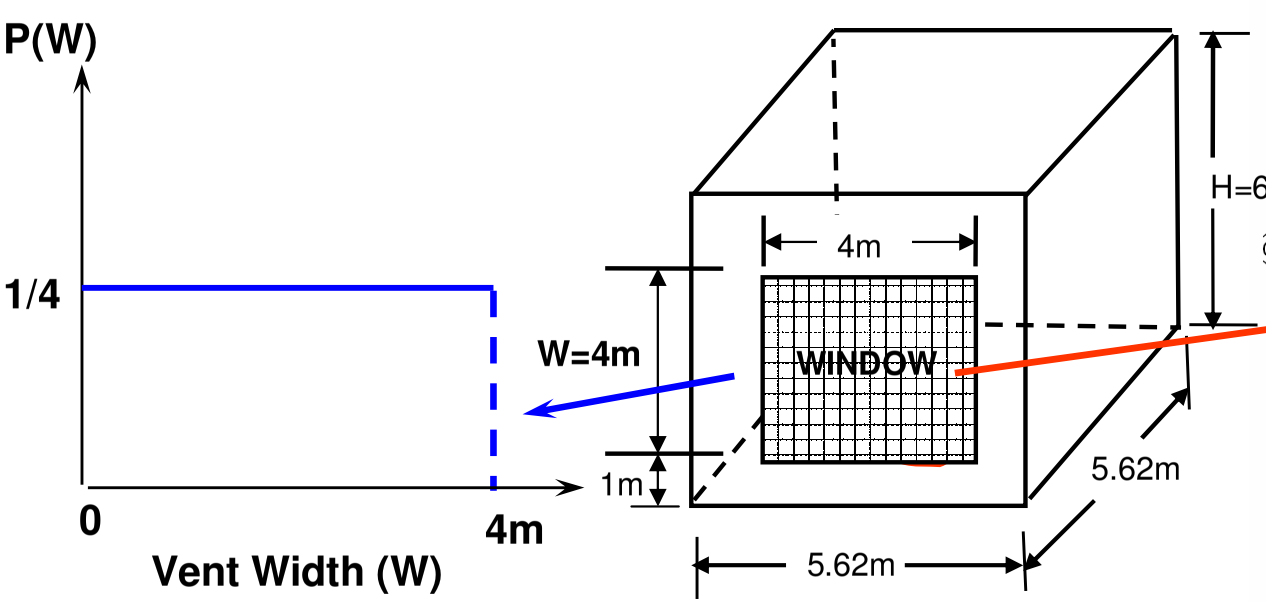
$$M_k^{(Z)} \approx \sum_{i=1}^{N_Q} \sum_{j=1}^{N_Q'} Z(\dot{S}_i,V_j)^k W_i^{(1)} W_j^{(2)}$$

CDF reconstruction using moment matching with a GLD*

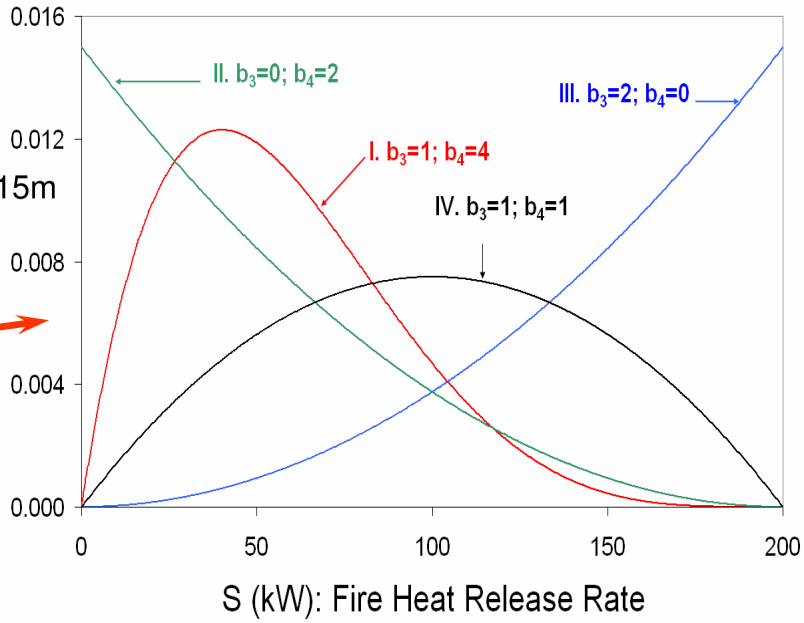


*Z. Karian & E. Dudewicz, Fitting Statistical Distributions...

Input PDFs for Fire Size and Vent Widths



Uniform Distribution for Vent Width



Beta Distributions for Fire Heat Release Rates

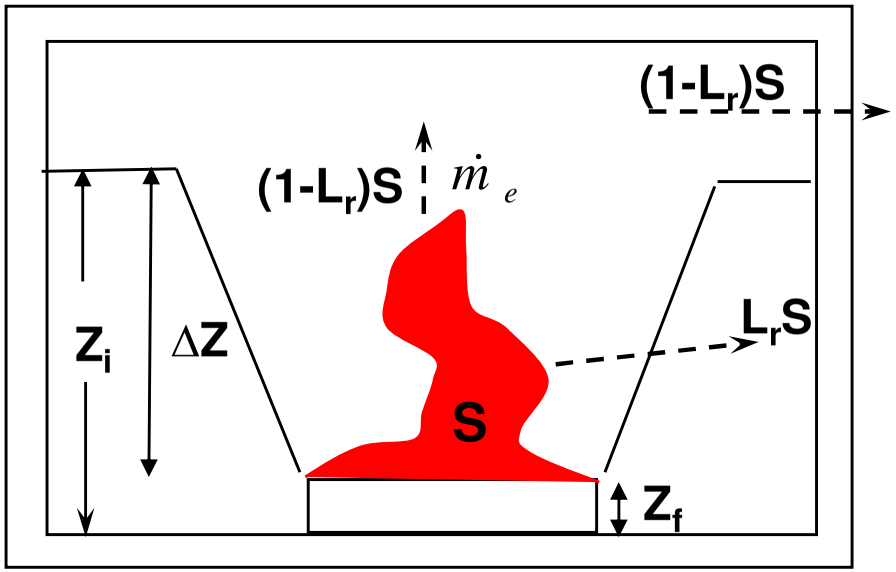
Fire Models: I. ASET Model*; II. CFAST

I. ASET (Available Safe Egress Time):

ASET Model gives $Z(\dot{S};t)$ as a solution of an ODE.

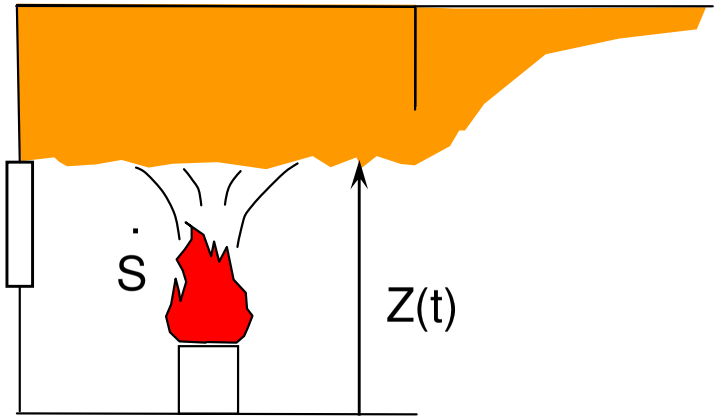
$$\frac{dZ}{dt} = -\frac{\dot{m}_e}{\rho_a A} - \frac{(1-L_c)\dot{S}}{\rho_a c_p T_a A}$$

$\dot{m}_e$: Rate of entrainment of air into the plume (Given by a correlation).



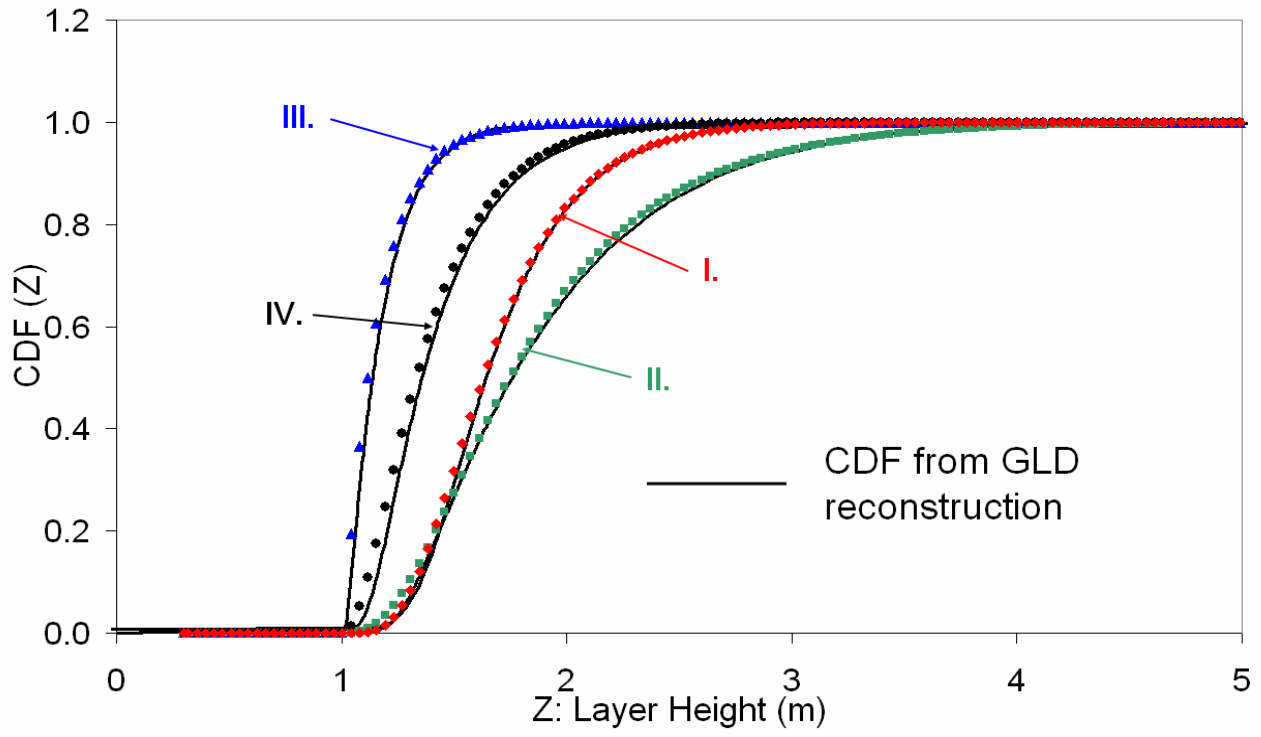
II. CFAST (Consolidated Fire and Smoke Transport):

- 2-Zone model that gives the smoke layer height as well as temperatures, smoke concentration and combustion gases.
- More detailed model involving solution of 4 coupled ODEs.



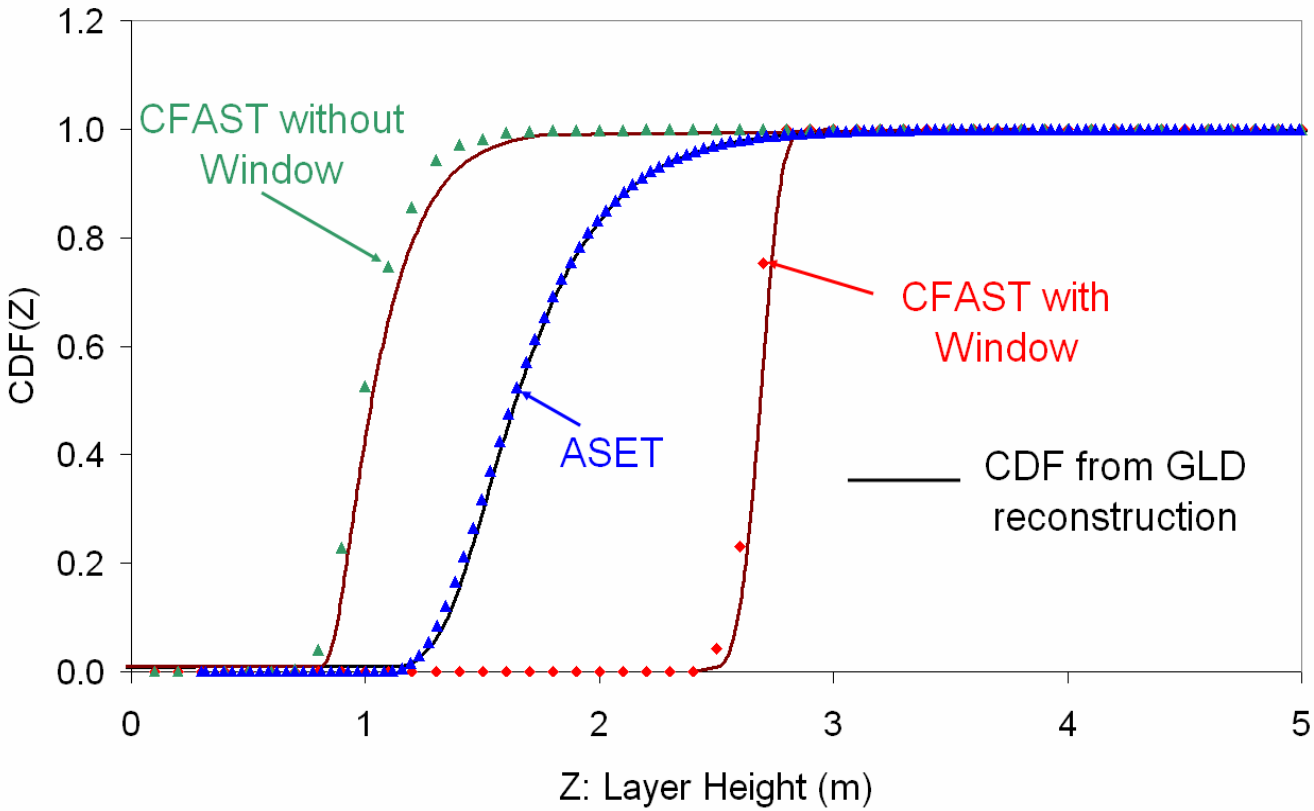
*Janssens,M.L. (2000), An Introduction to Mathematical Fire Modelling.

Results from the QMOM and CDF reconstruction : ASET model



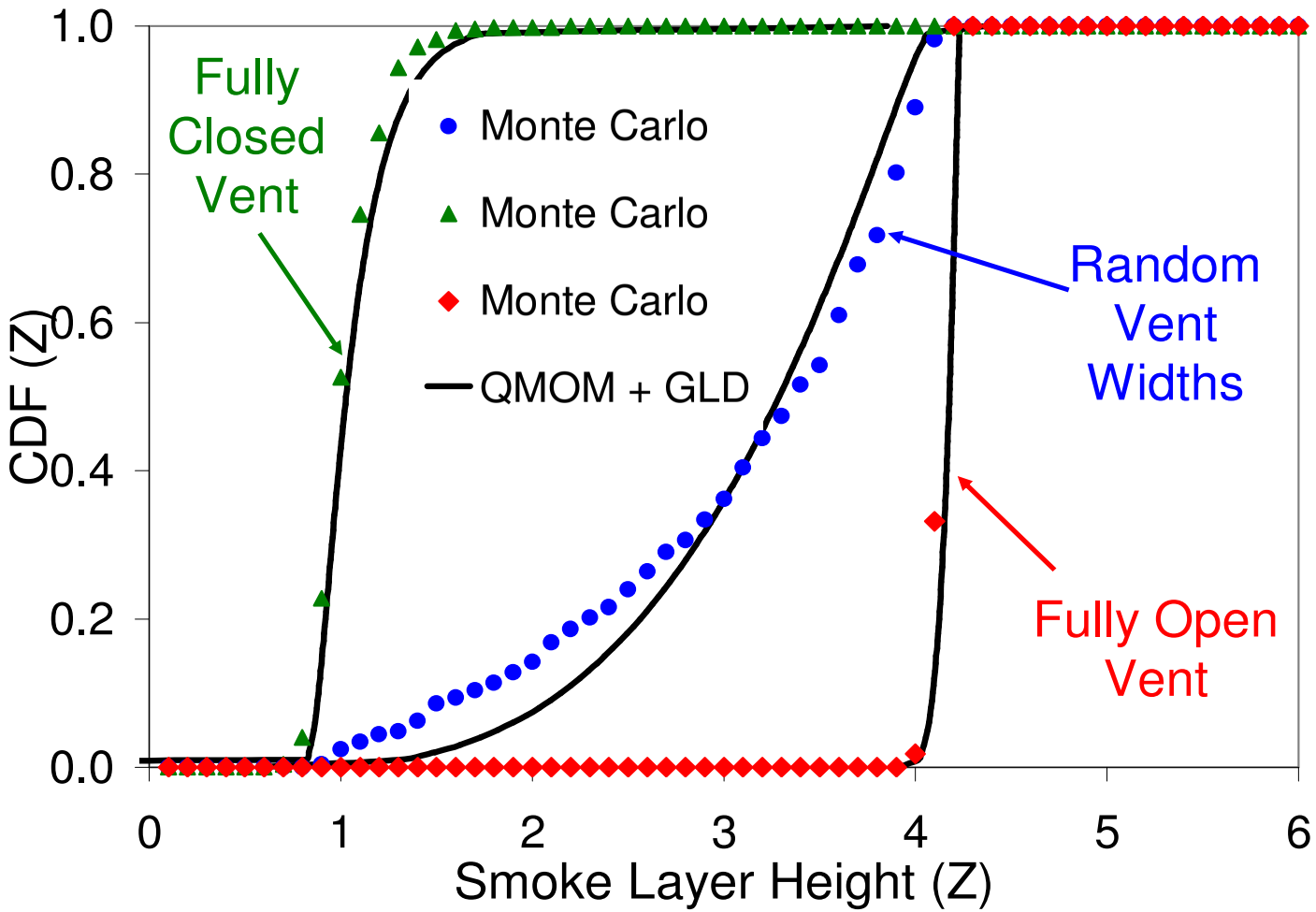
- Reconstructed CDF by matching moments of Z with those of the Generalized Lambda Distribution (GLD), univariate case.
- 3 Quadrature points chosen to get the four moments needed for the reconstruction.
- CDF prediction is remarkably accurate for the extremely small number of samples used.

Results from the QMOM and CDF reconstruction : CFAST



- Comparison of CDF obtained from Monte Carlo simulations with the CDF obtained from Method of Moments and GLD reconstruction, univariate case.
- 3 Quadrature points chosen to get the four moments for the GLD reconstruction.
- GLD provides an accurate reconstruction for all cases.

Results for the Bivariate Case: CFAST



- Figure shows the CDF for different vent conditions.

- QMOM with GLD reconstruction gives a satisfactory estimate of the CDF with only 3 samples!

Conclusions

- Uncertainty is an intrinsic aspect of fire and must be considered in designing for fire safety.
- Use of Monte Carlo simulations in Performance Based Fire Designs can be computationally expensive.
- The Quadrature Method of Moments reduces computational costs by solving for only the moments of the PDF.
- A moment matching scheme was used to reconstruct the CDF needed for risk assessment.
- Estimation of the CDF using moment matching was shown to be fairly accurate for a number of scenarios.